Modern Channel Coding

Mini-Module 5:

Symmetric Channels, Information Combining, Decoding Thresholds for LDPC Codes

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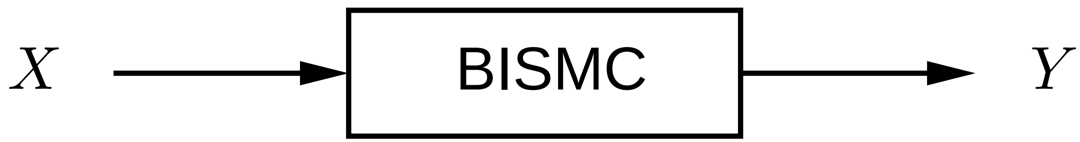
Aalborg University, 6 -10 June 2005

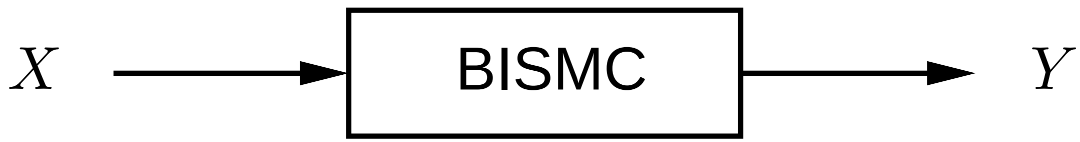
Symmetric Channels



Binary-Input Symmetric Memoryless Channel

- binary input alphabet: $X \in \{+1, -1\}$
- arbitrary output alphabet: $Y \in \mathbb{R}$
- memoryless
- symmetric: $p(y|x) = p(-y| - x)$

Symmetric Channels



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> Channel can be decomposed into strongly symmetric sub-channels

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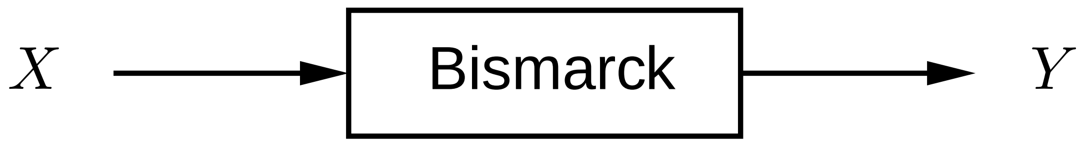

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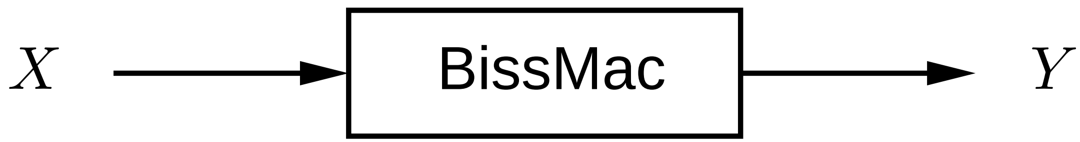

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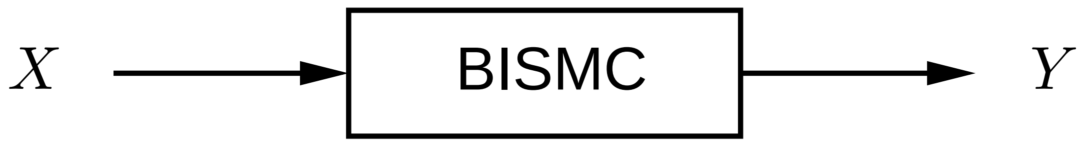

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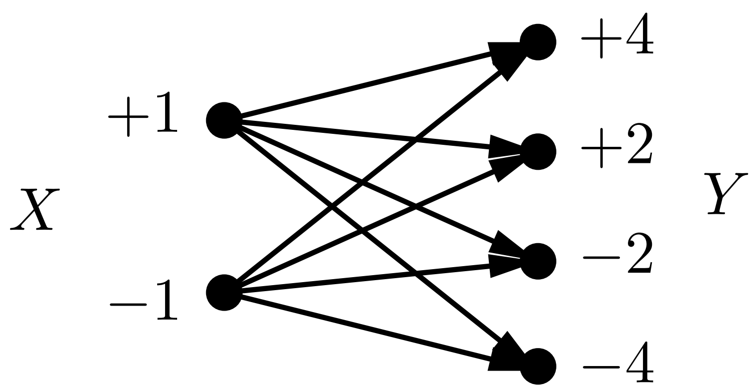
Example: BISMC

Original Channel

Sub-Channels



Example: BISMC

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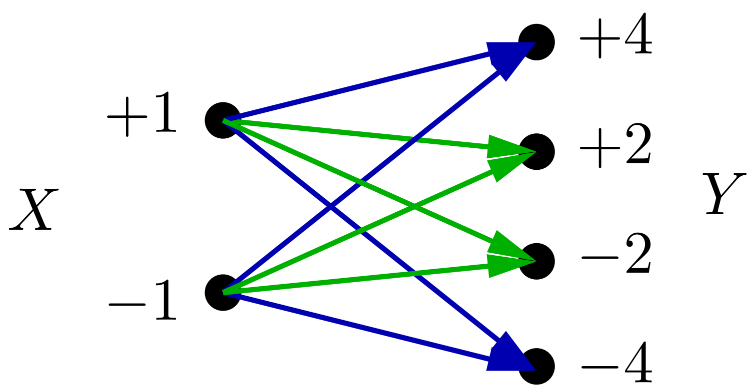
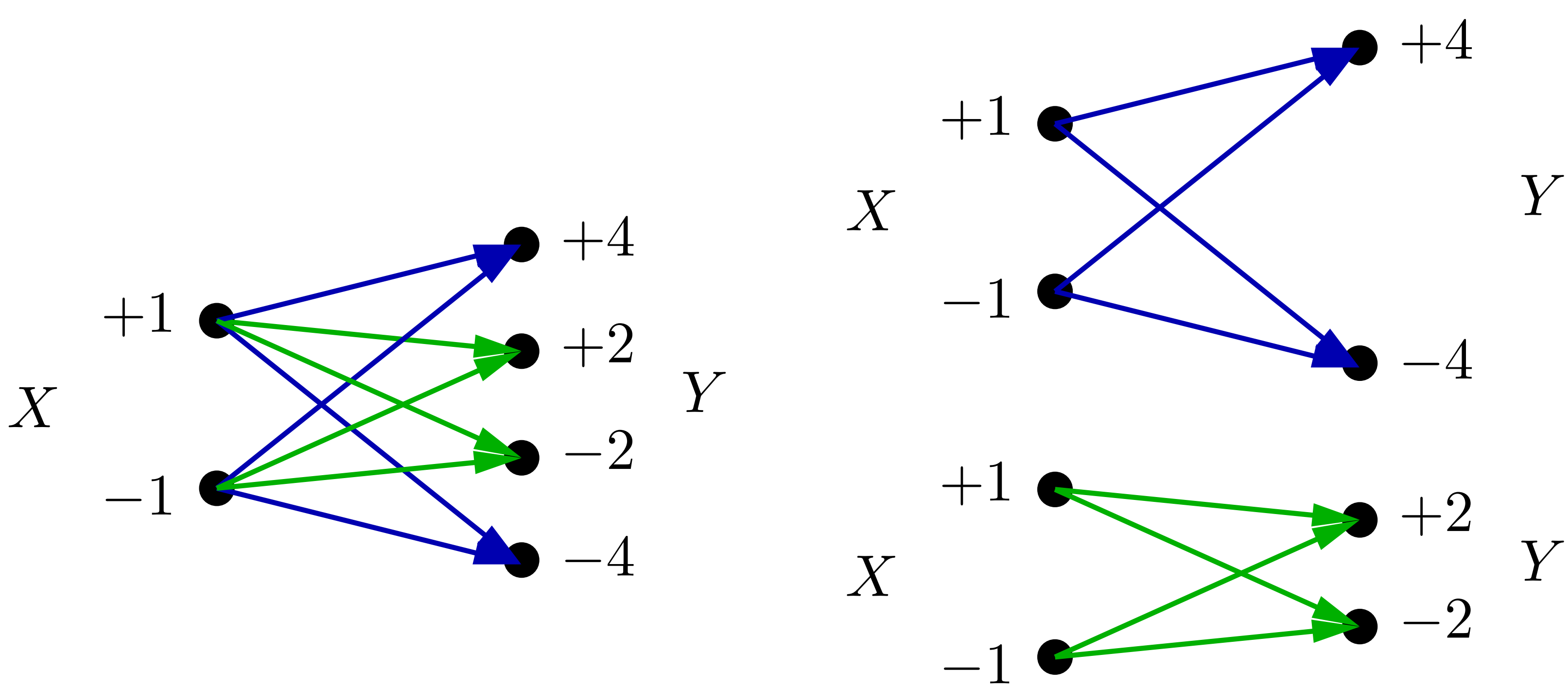


Example: BISMC

Original Channel　　　　Sub-Channels



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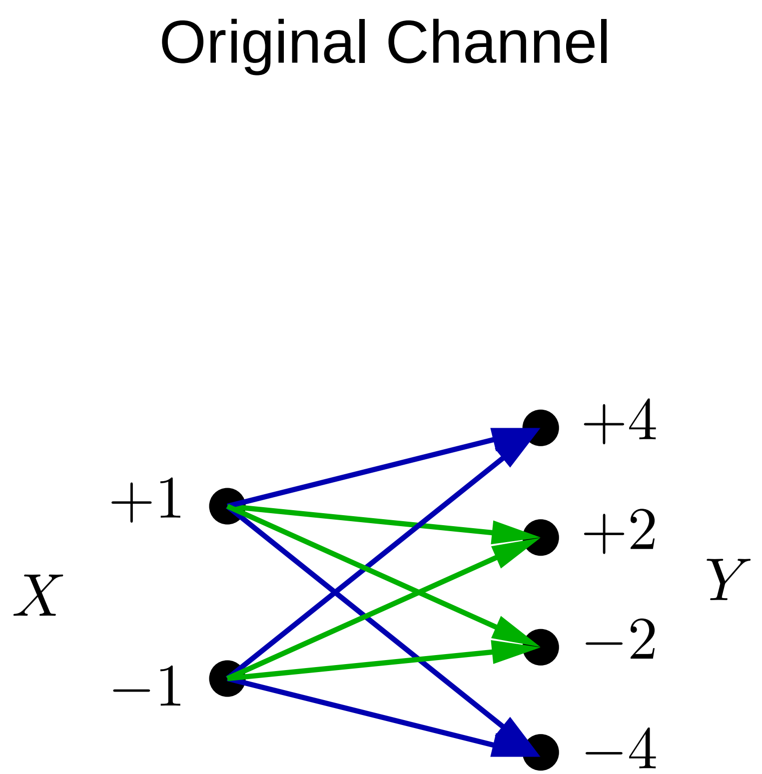
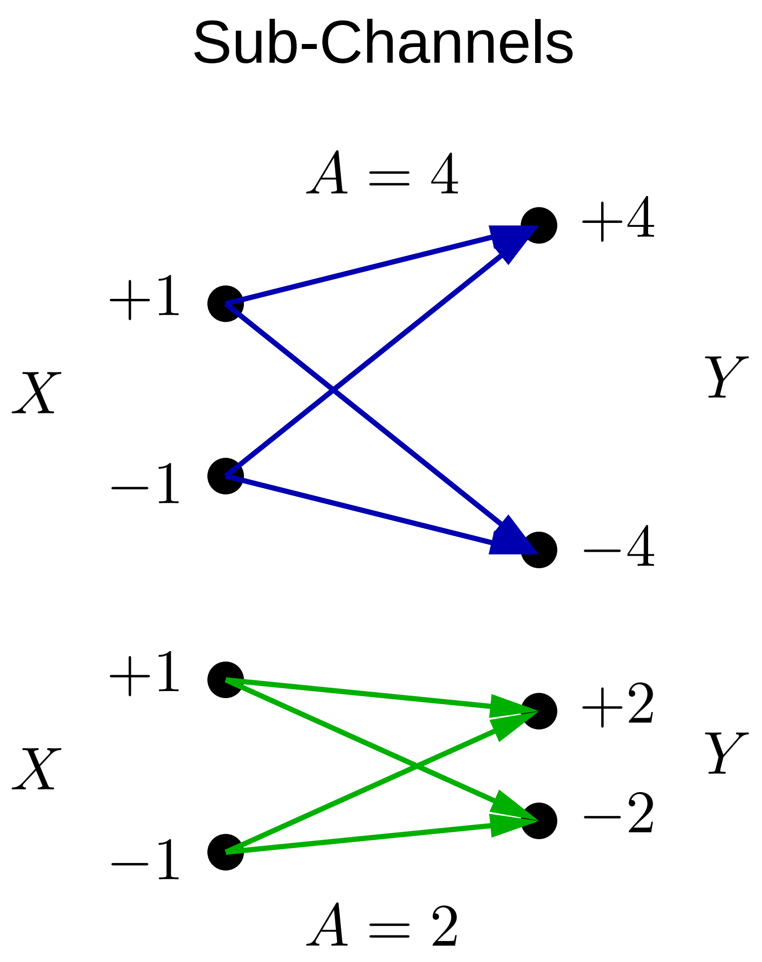
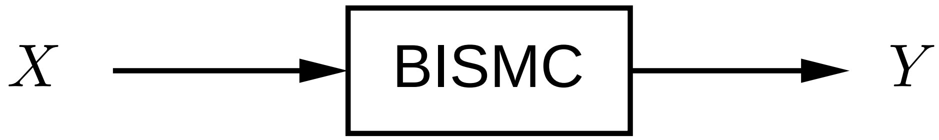




- Sub-channel indicator: $A := |Y|$

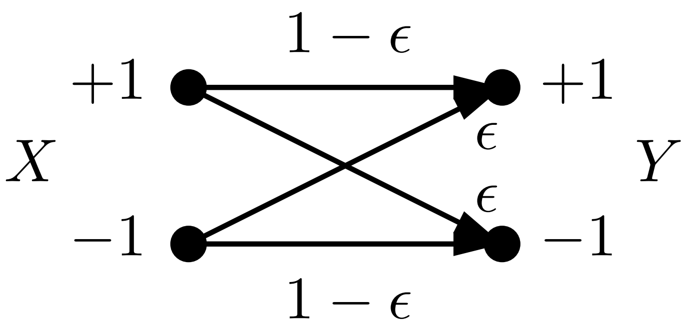
Mutual Information



- **BISMC** with transition probabilities $p_{Y|X}(y|x)$

$$I(X;Y) := \mathrm{E}\left\{\log_2 \frac{p_{X|Y}(x|y)}{p_X(x)}\right\}$$

Mutual Information



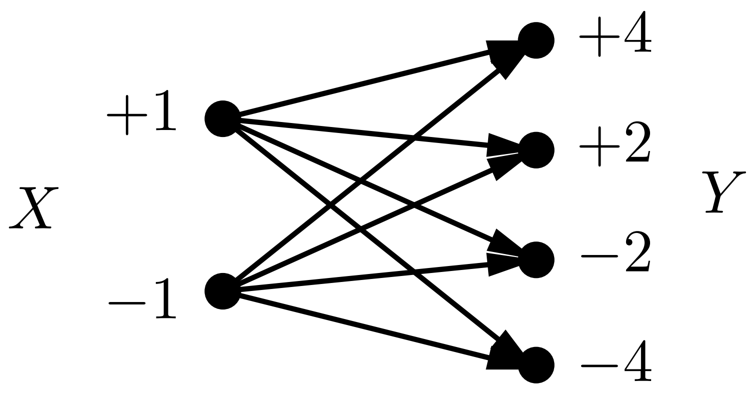
- **BSC** with crossover probability ϵ

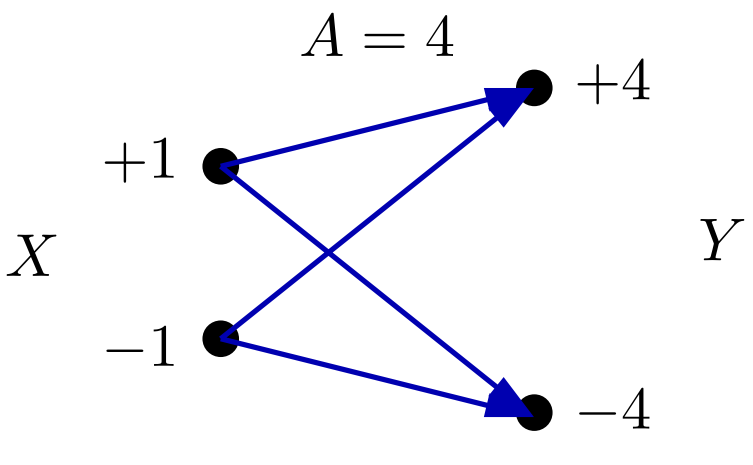
$$I(X;Y) := 1 - h(\epsilon)$$

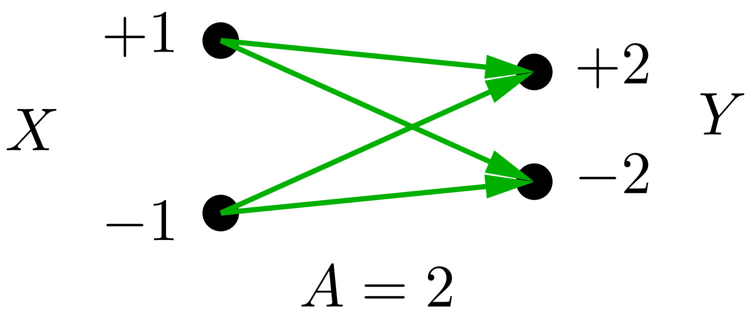
- Binary entropy function

$$h(\epsilon) := -\epsilon \log_2 \epsilon - (1-\epsilon)\log_2(1-\epsilon)$$

Example: BISMC (2)







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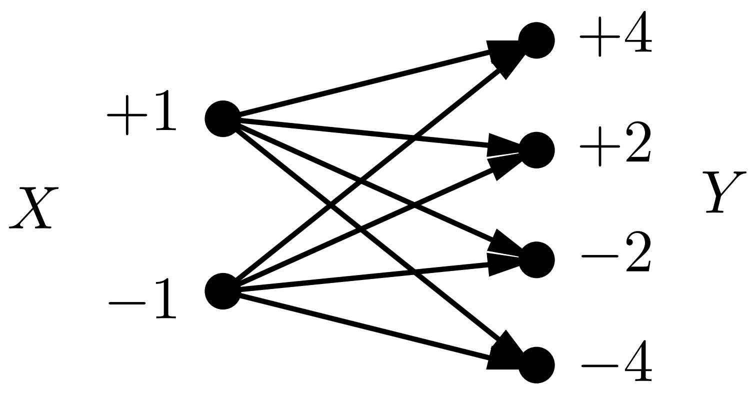
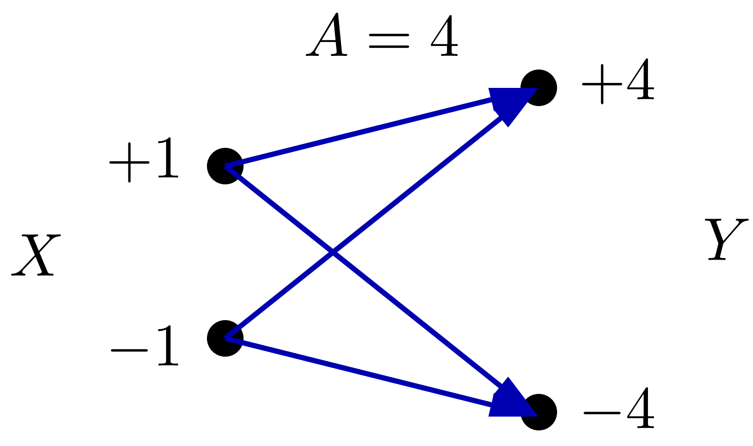
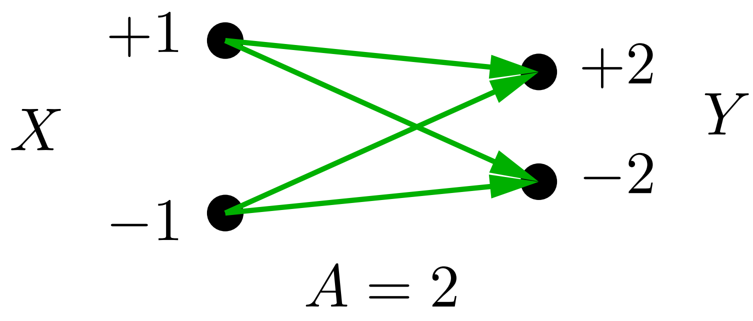






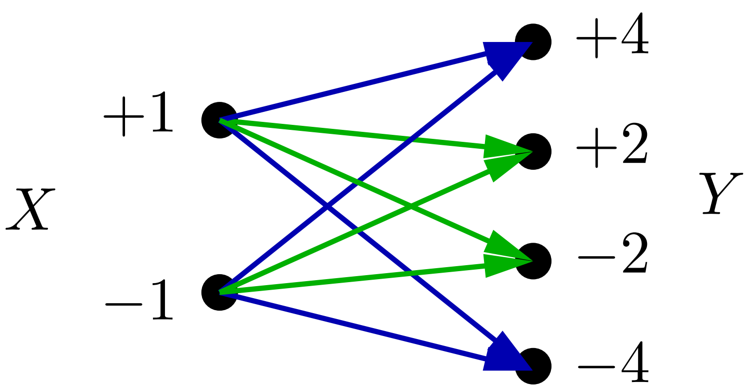
(Remember: $A := |Y|$)

- $I(X; Y)$

- $I(X; Y | A = 4) = 1 - h(\epsilon_1)$
- $I(X; Y | A = 2) = 1 - h(\epsilon_2)$

Example: BISMC

Decomposition into
sub-channels that are BSCs



Example: BISMC

- Mutual information of blue sub-channel:

  $$j_1 = 1 - h(\epsilon_1)$$

  of green sub-channel:

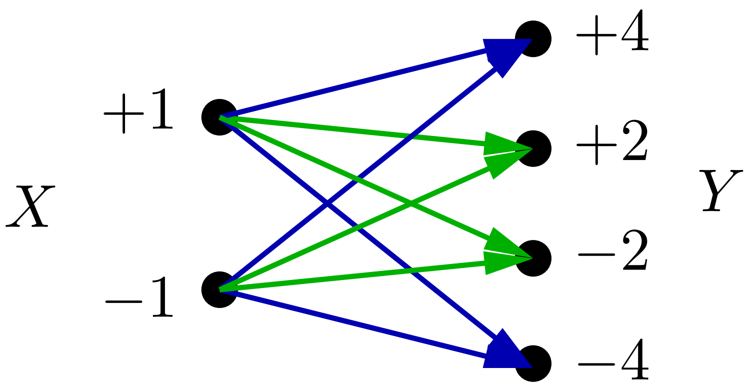
  $$j_2 = 1 - h(\epsilon_2)$$

  General: $J \in \{j_1, j_2\}$

- Mutual information of (original) channel

  $$I(X;Y) = \mathrm{E}\{J\}$$

Decomposition into sub-channels that are BSCs



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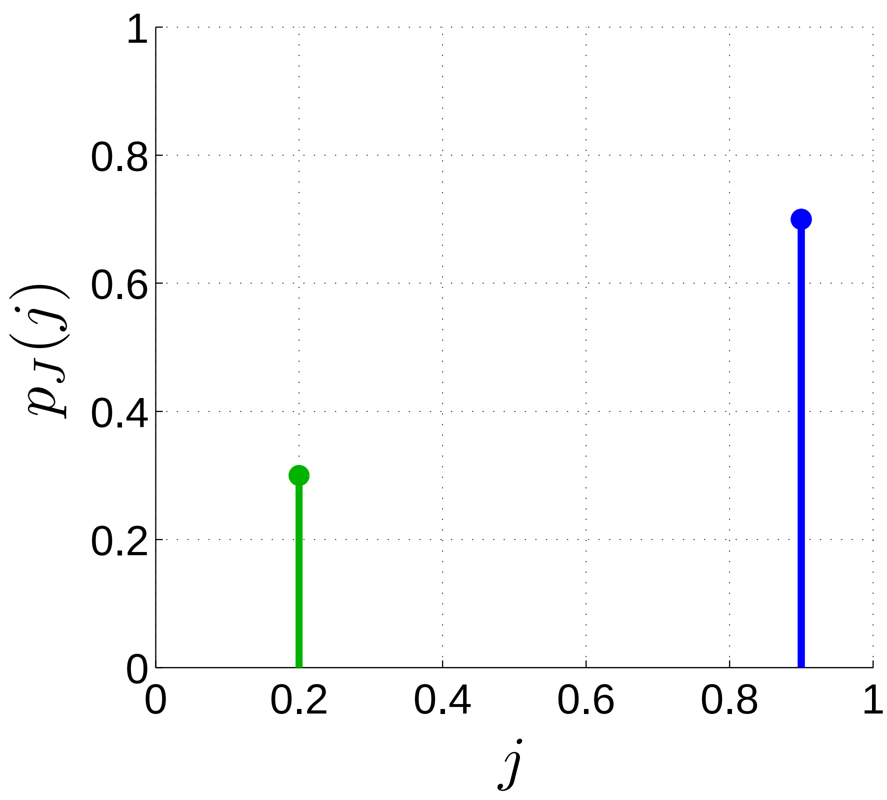
Mutual Information Profile



Mutual Information Profile

- Binary-input symmetric memoryless channel
- **Property of a BISMC** (also for continuous output alphabet)

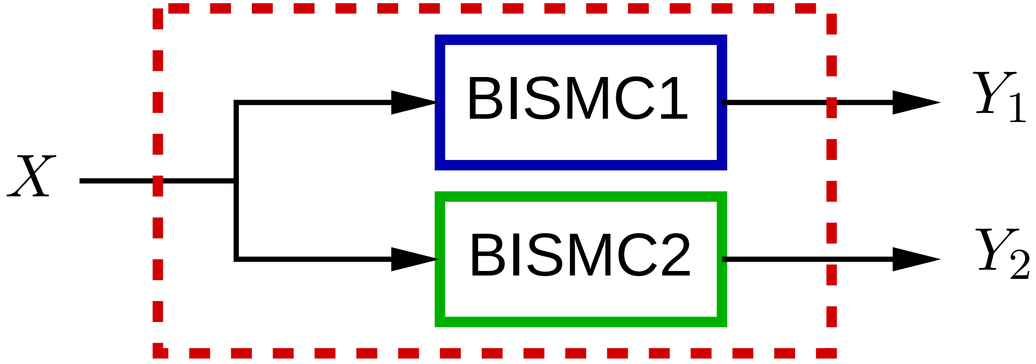
  > Decomposable into sub-channels that are BSCs

- **Characterization of a BISMC**

  > Mutual Information Profile
  >
  > Distribution $p_J(j)$ of the sub-channel mutual information J

- **Notice**: $I(X;Y) = \mathrm{E}\{J\}$

Information Combining



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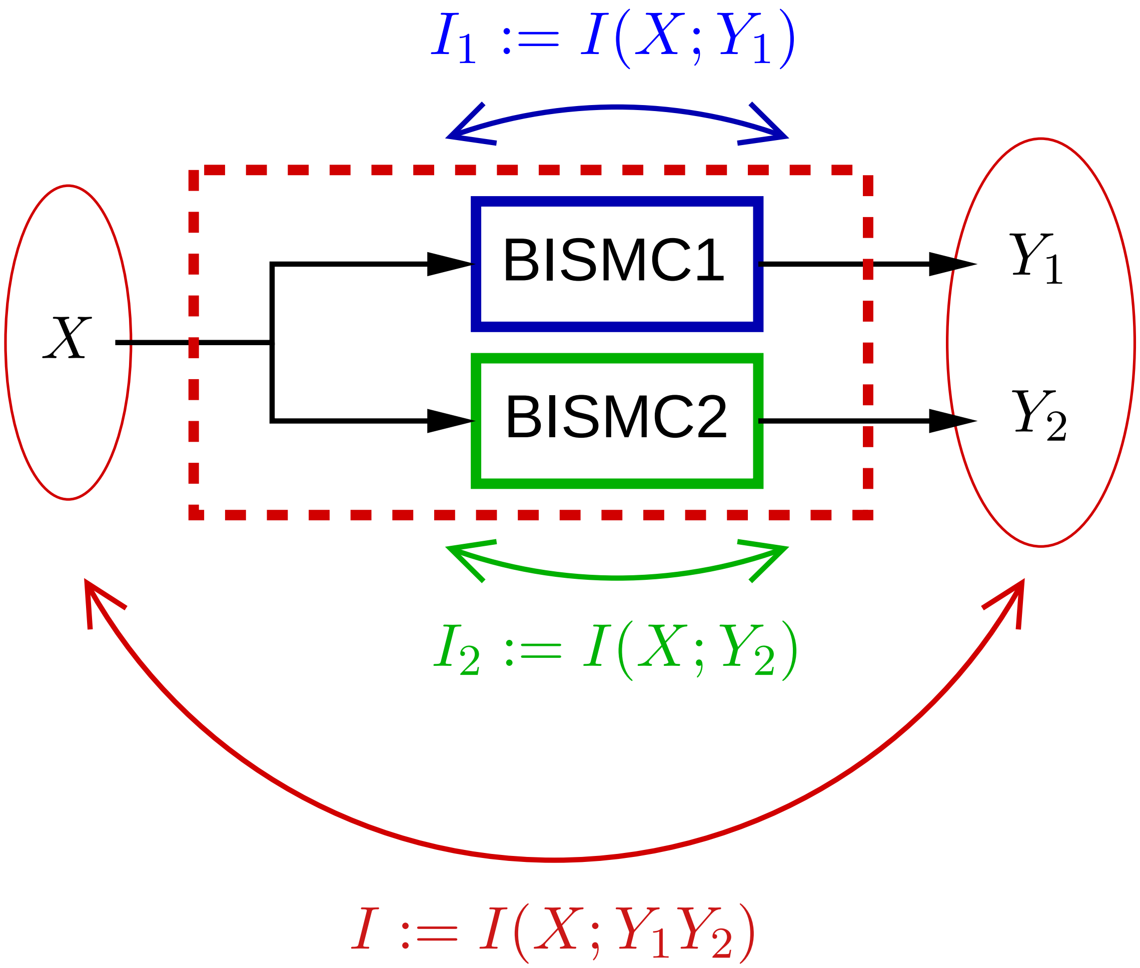
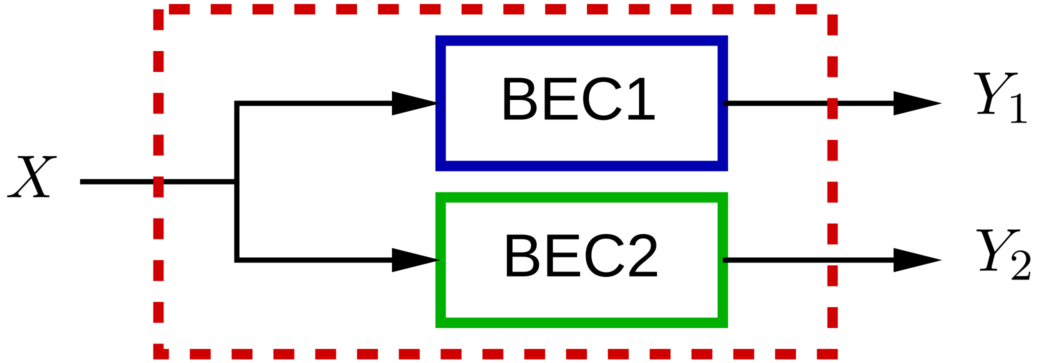


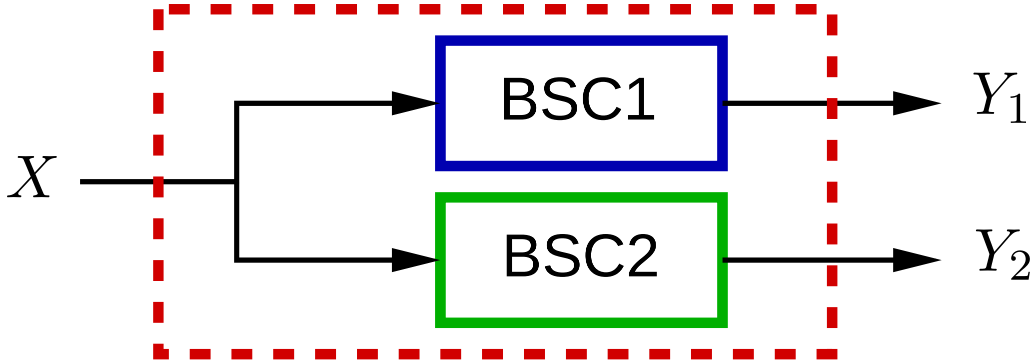
Examples for Information Combining



- **Binary Erasure Channels**

$$I = 1 - (1 - I_1)(1 - I_2)$$

Examples for Information Combining

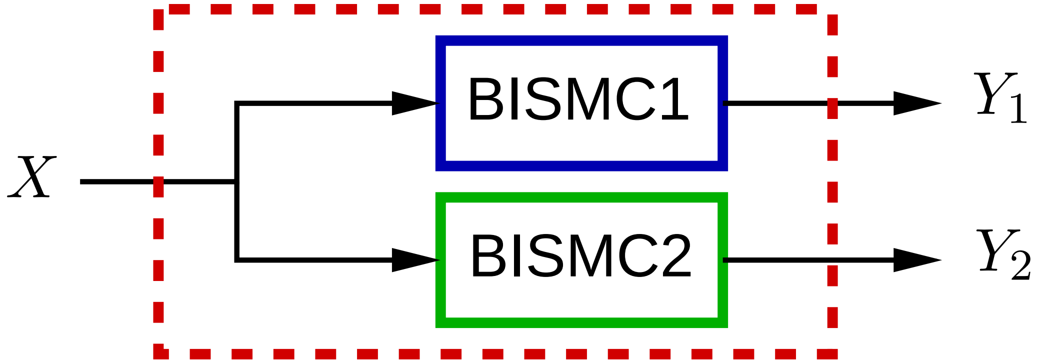


- **Binary Symmetric Channels**

$$I = f_2^{\text{par}}(I_1, I_2)$$

with $f_2^{\text{par}}(I_1, I_2) := I_1 + I_2 - 1 + h\Big(\epsilon_1(1-\epsilon_2) + (1-\epsilon_1)\epsilon_2\Big)$,
$\epsilon_1 = h^{-1}(1 - I_1)$, $\epsilon_2 = h^{-1}(1 - I_2)$

Information Combining: Problem



- Overall mutual information depends on channel models
- Bounds on overall mutual information?

Information Combining: Solution (1)

$$
\begin{aligned}
I &:= I(X;Y_1Y_2) \\
&= I(X;Y_1Y_2|J_1J_2) \\
&= \int_{j_1}\int_{j_2} \underbrace{p_{J_1}(j_1)}_{\text{MIP1}}\,\underbrace{p_{J_2}(j_2)}_{\text{MIP2}}\,\underbrace{I(X;Y_1Y_2|J_1=j_1,J_2=j_2)}_{2\times\text{BSC}}\;dj_1\,dj_2 \\
&= \mathrm{E}\Big\{\underbrace{f_2^{\text{par}}(J_1,J_2)}_{2\times\text{BSC}}\Big\}
\end{aligned}
$$

Information Combining: Solution (2)

- Bounding the expected value:

$$\mathrm{E}\Big\{ f_2^{\mathsf{par}}(J_1, J_2) \Big\} \quad \geq$$

$$\mathrm{E}\Big\{ f_2^{\mathsf{par}}(J_1, J_2) \Big\} \quad \leq$$

- Properties of $f_2^{\mathsf{par}}(j_1, j_2)$:

Information Combining: Solution (2)

- Bounding the expected value:

$$\mathrm{E}\Big\{ f_2^{\text{par}}(J_1, J_2) \Big\} \;\geq\; f_2^{\text{par}}\Big(\mathrm{E}\{J_1\}, \mathrm{E}\{J_2\}\Big) = f_2^{\text{par}}(I_1, I_2)$$

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  - $f_2^{\text{par}}(j_1, j_2)$ is convex-$\cup$ (Mrs. Gerber's Lemma)

Information Combining: Solution (2)

- Bounding the expected value:

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$$\mathrm{E}\Big\{f_2^{\mathsf{par}}(J_1, J_2)\Big\} \leq \mathrm{E}\Big\{1 - (1 - J_1)(1 - J_2)\Big\} = $$

$$= 1 - (1 - I_1)(1 - I_2)$$

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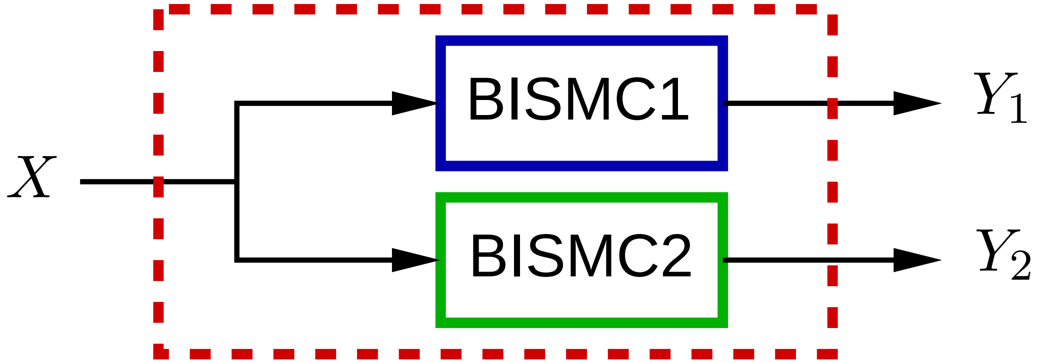
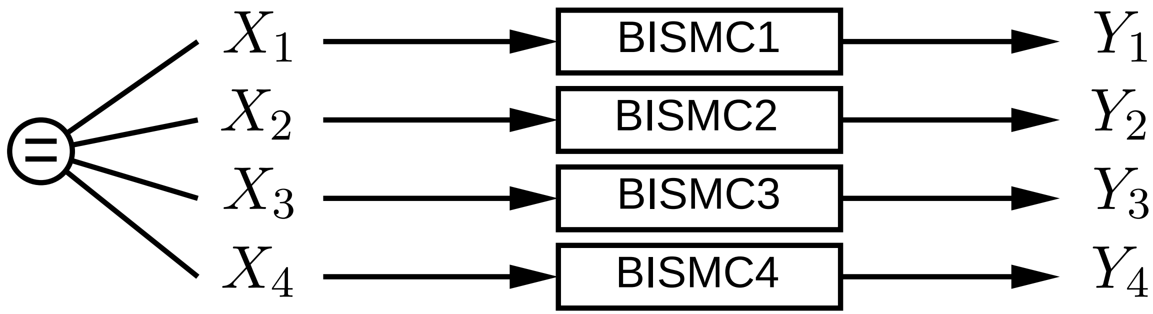
Information Combining: Bounds



- Mutual information of the component channels $I_1 := I(X;Y_1)$ and $I_2 := I(X;Y_2)$
- Mutual information of the overall channel $I := I(X;Y_1 Y_2)$

$$\underbrace{f_2^{\text{par}}(I_1, I_2)}_{2 \times \text{BSC}} \leq I \leq \underbrace{1 - (1 - I_1)(1 - I_2)}_{2 \times \text{BEC}}$$

Repeat Codes



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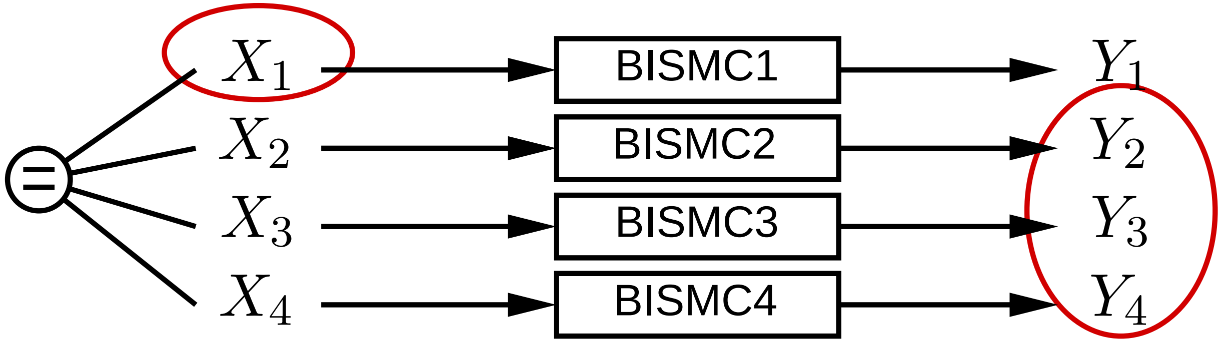

- **Given**:
  Mutual informations of the component channels

$$I_n := I(X_n; Y_n), \qquad n = 1, 2, 3, 4$$

- **Wanted**:
  Bounds on the extrinsic mutual information

$$I_{\mathrm{e},1} := I(X_1; Y_2 Y_3 Y_4)$$

(analogous for X_2, X_3, X_4)

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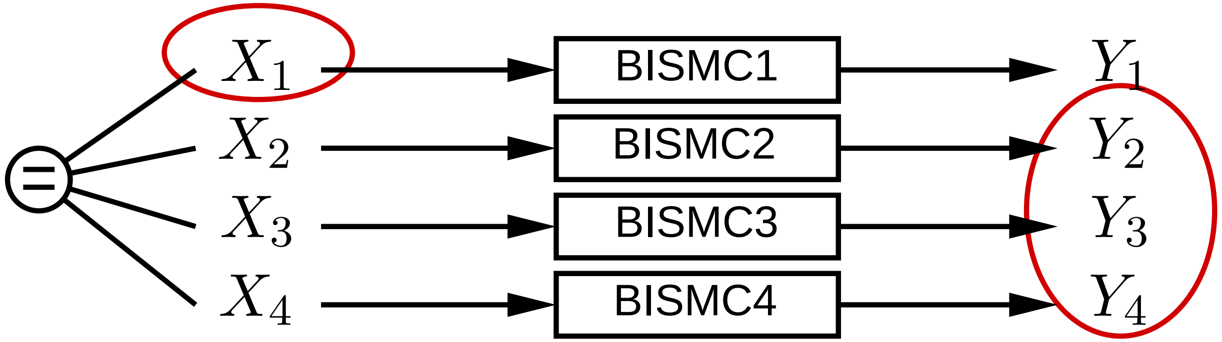

- **Solution**:

$$\underbrace{f_3^{\text{par}}(I_2, I_3, I_4)}_{\text{only BSCs}} \leq I_{\text{e},1} \leq \underbrace{1-(1-I_2)(1-I_3)(1-I_4)}_{\text{only BECs}}$$

Repetition Codes: EXIT Functions

- Code length N
- Input:
  $I_n := I_{\mathsf{a}} \quad \forall n$
- Output:
  $I_{\mathsf{e}} := \frac{1}{N} \sum_{n=1}^{N} I_{\mathsf{e},n}$
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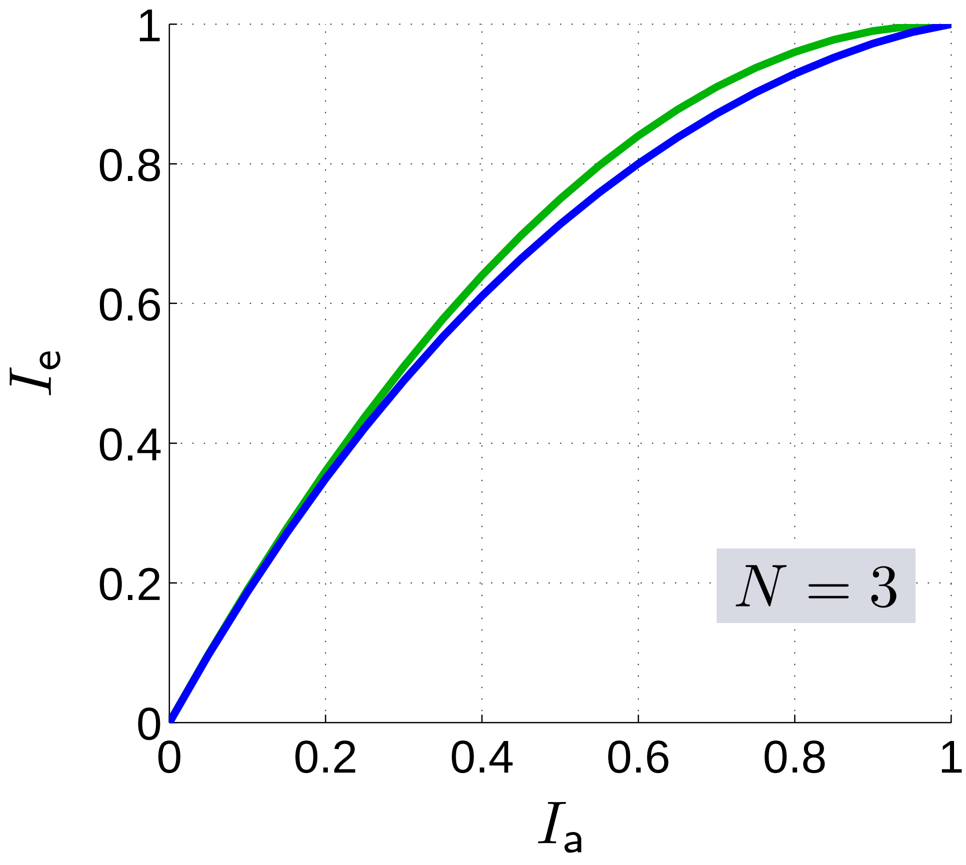

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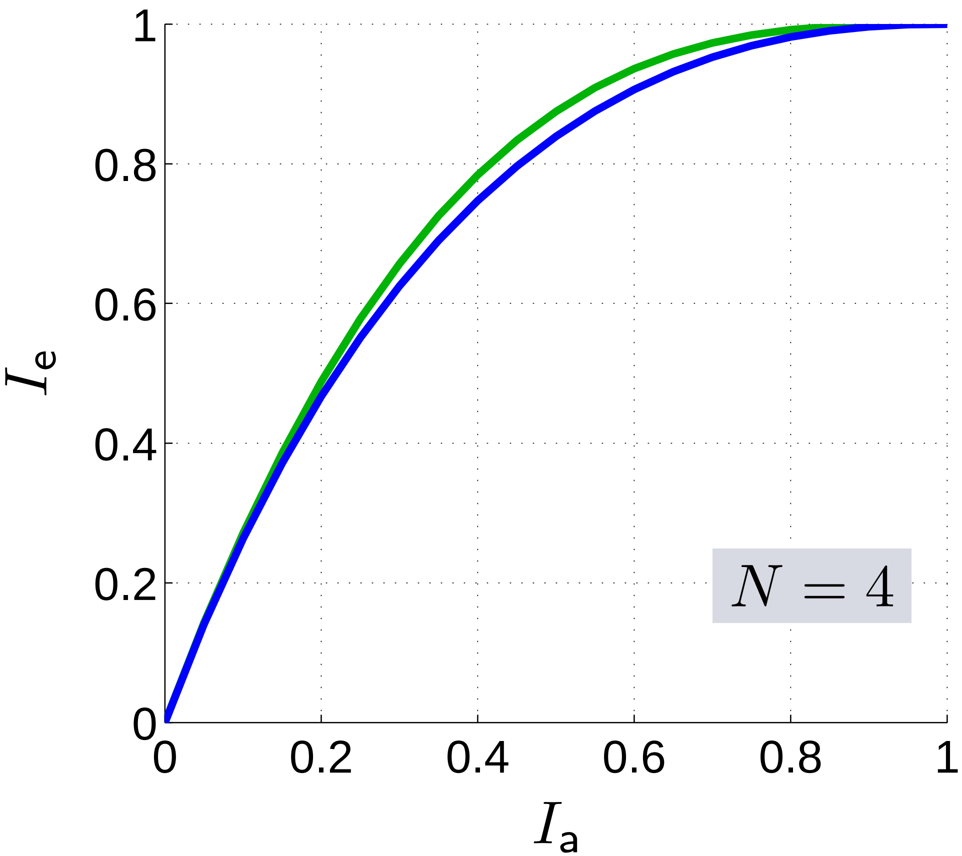

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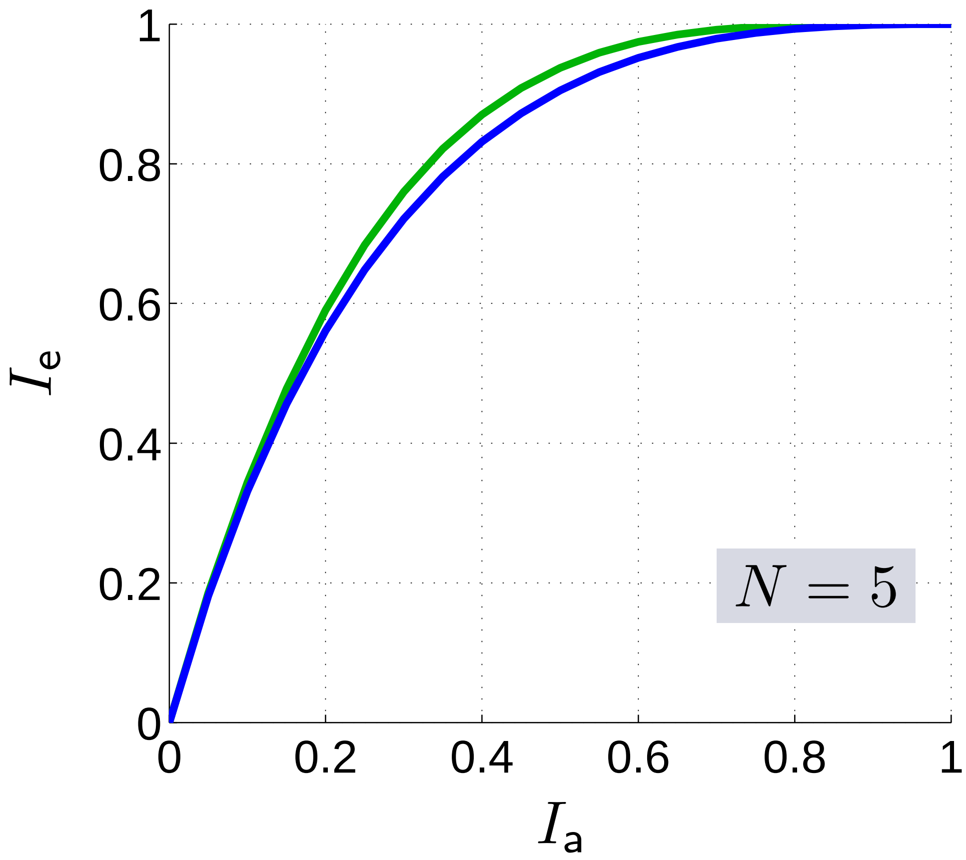

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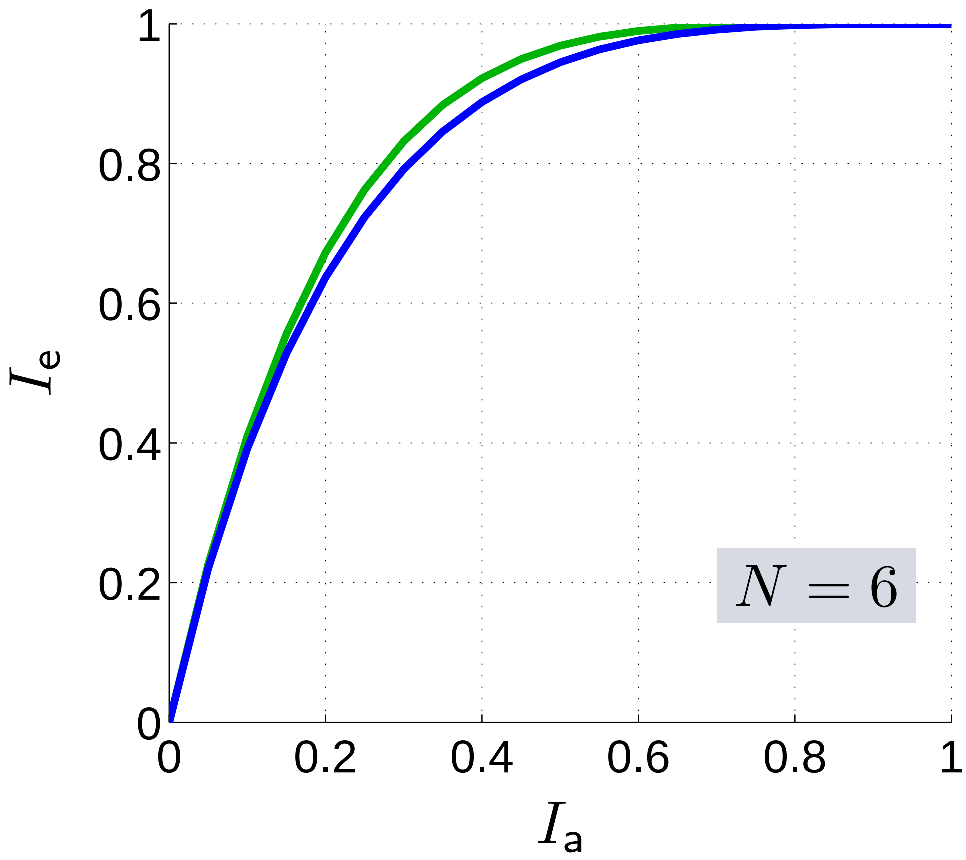

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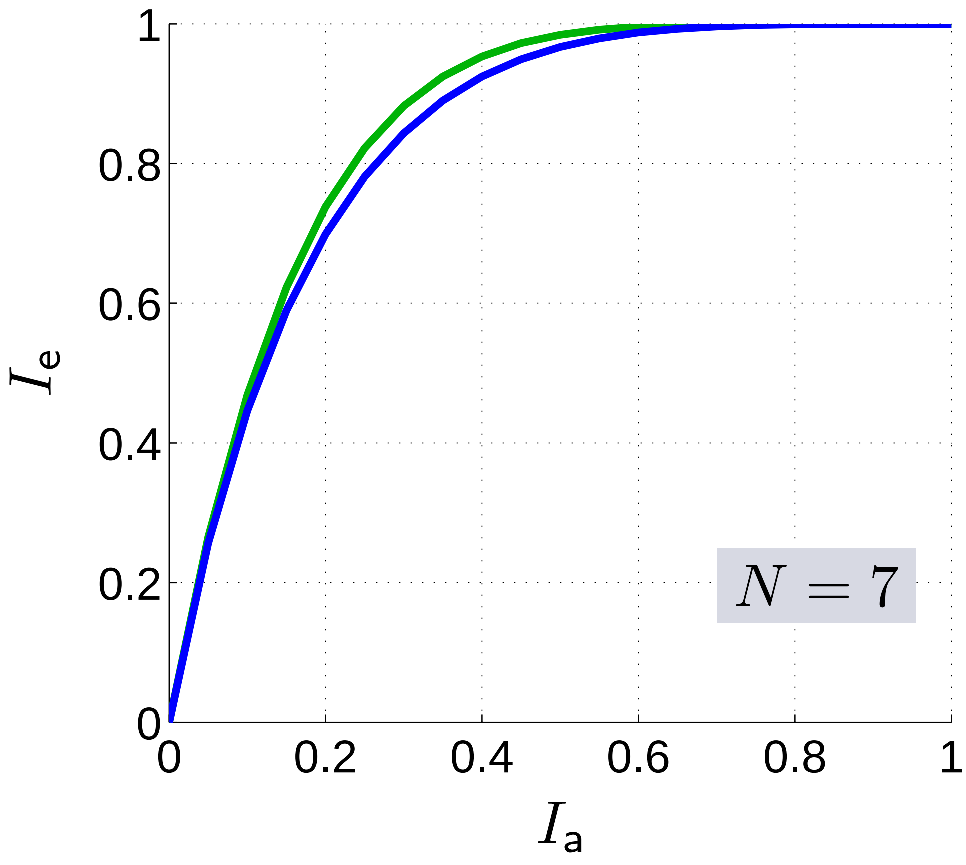

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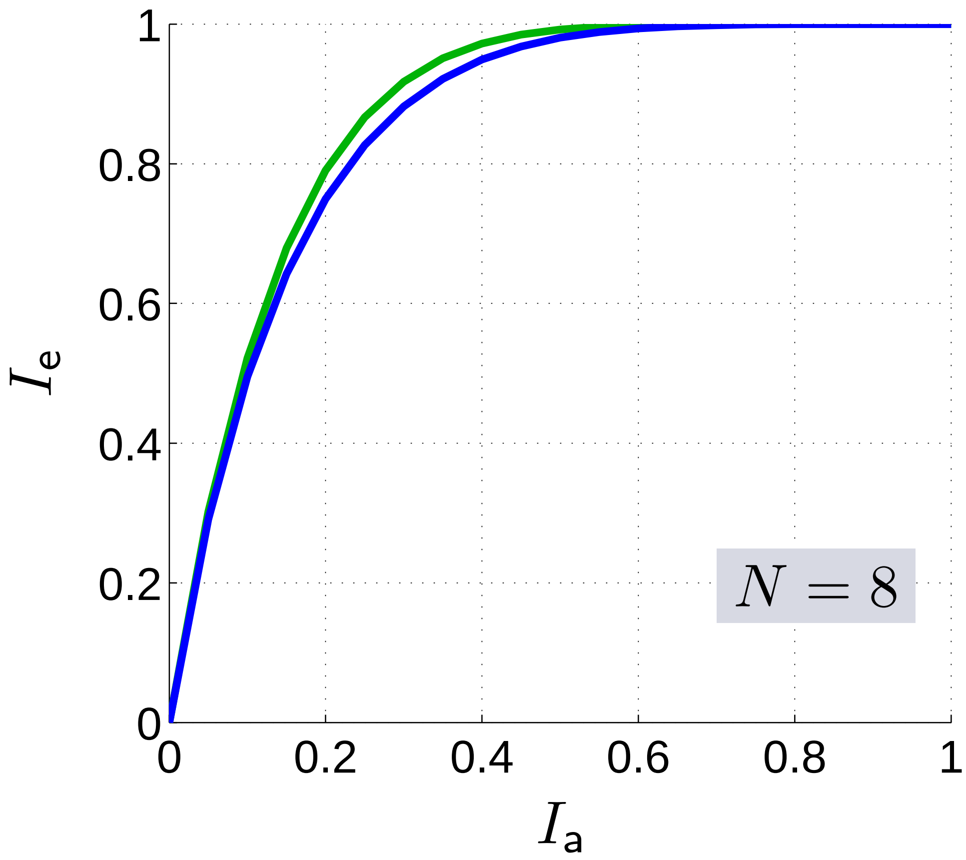

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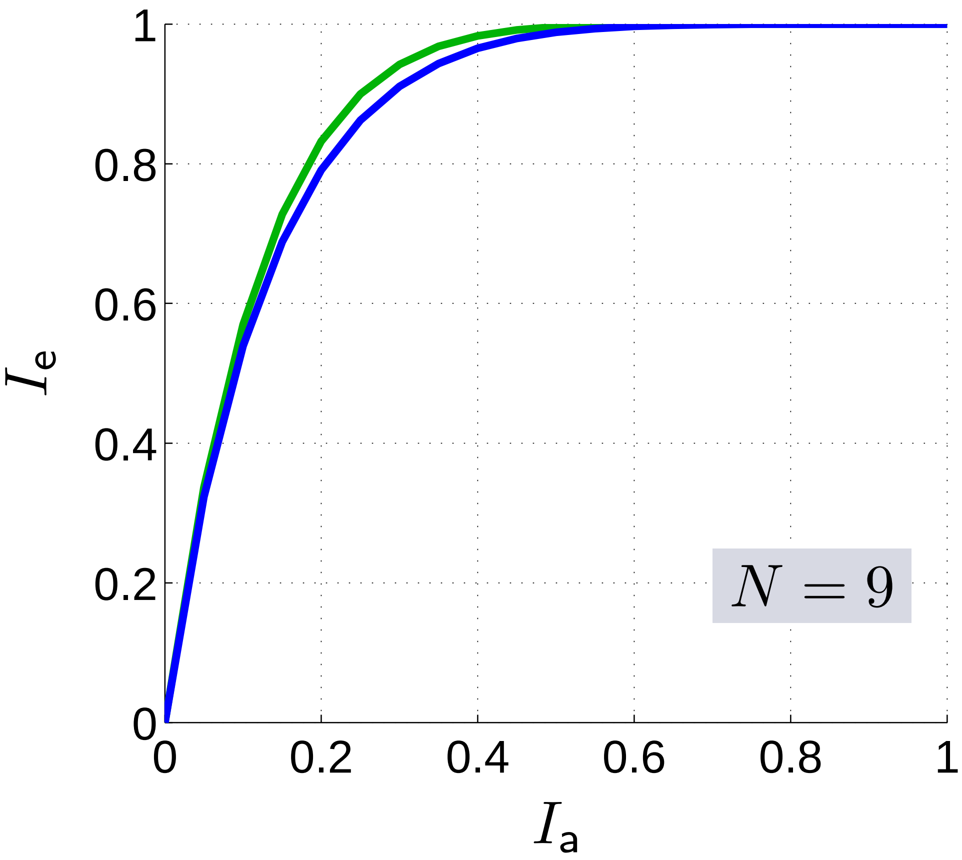
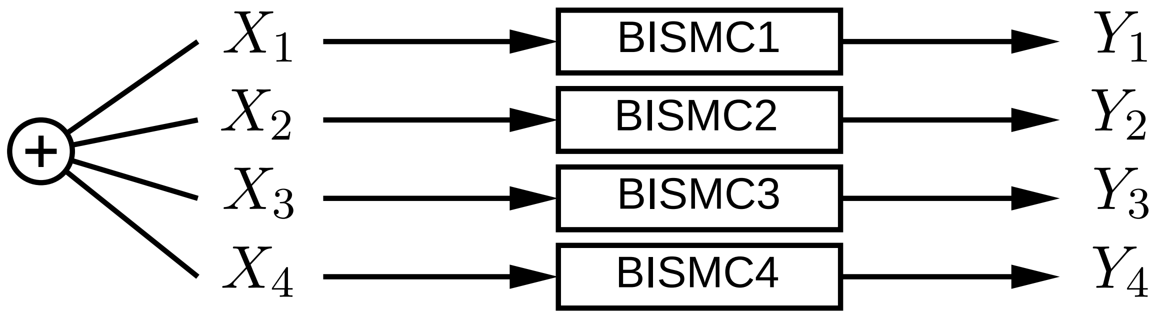


Single Parity-Check Codes



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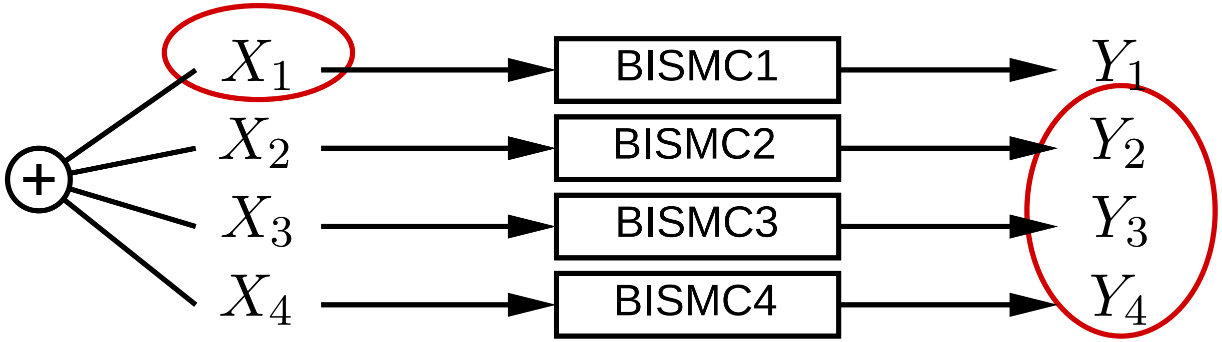

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- **Wanted**:
  Bounds on the extrinsic mutual information

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(similarly for X_1, X_2, X_3)

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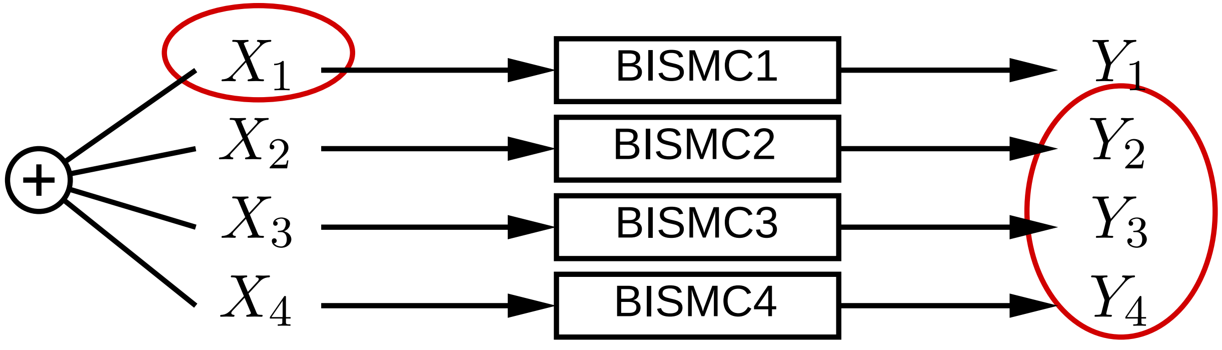

- **Solution**:

$$\underbrace{I_2 \cdot I_3 \cdot I_4}_{\text{only BECs}} \quad \leq \quad I_{\mathrm{e},1} \quad \leq \quad \underbrace{f_3^{\mathrm{ser}}(I_2, I_3, I_4)}_{\text{only BSCs}}$$

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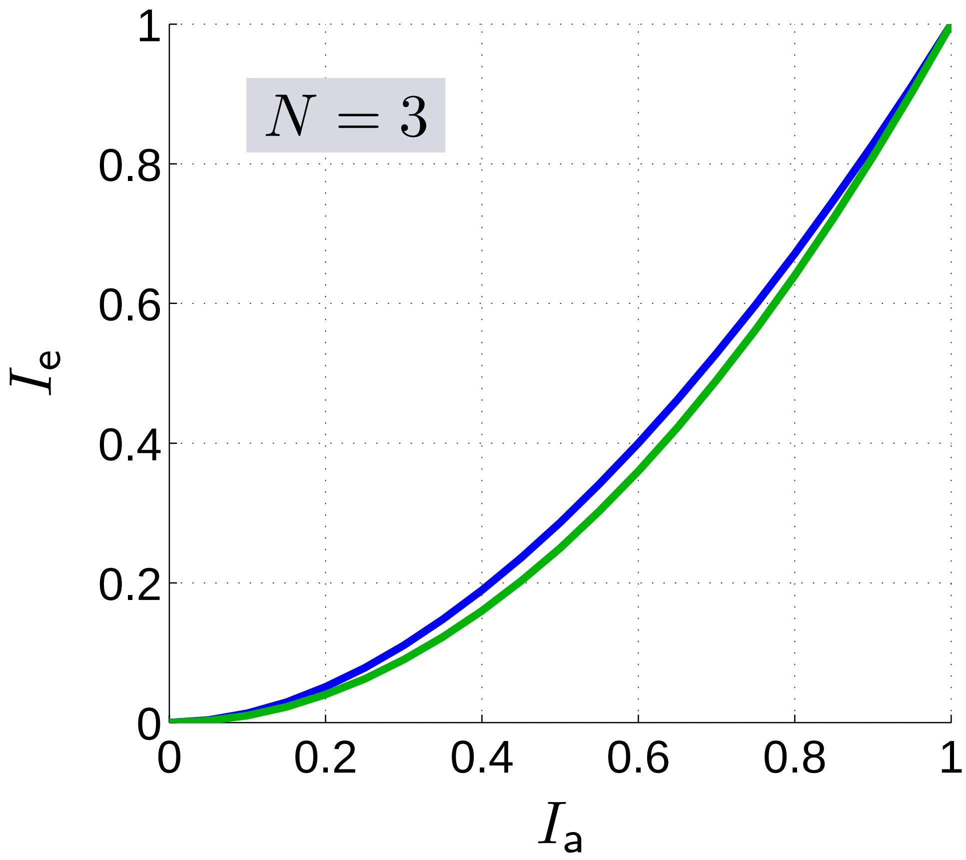

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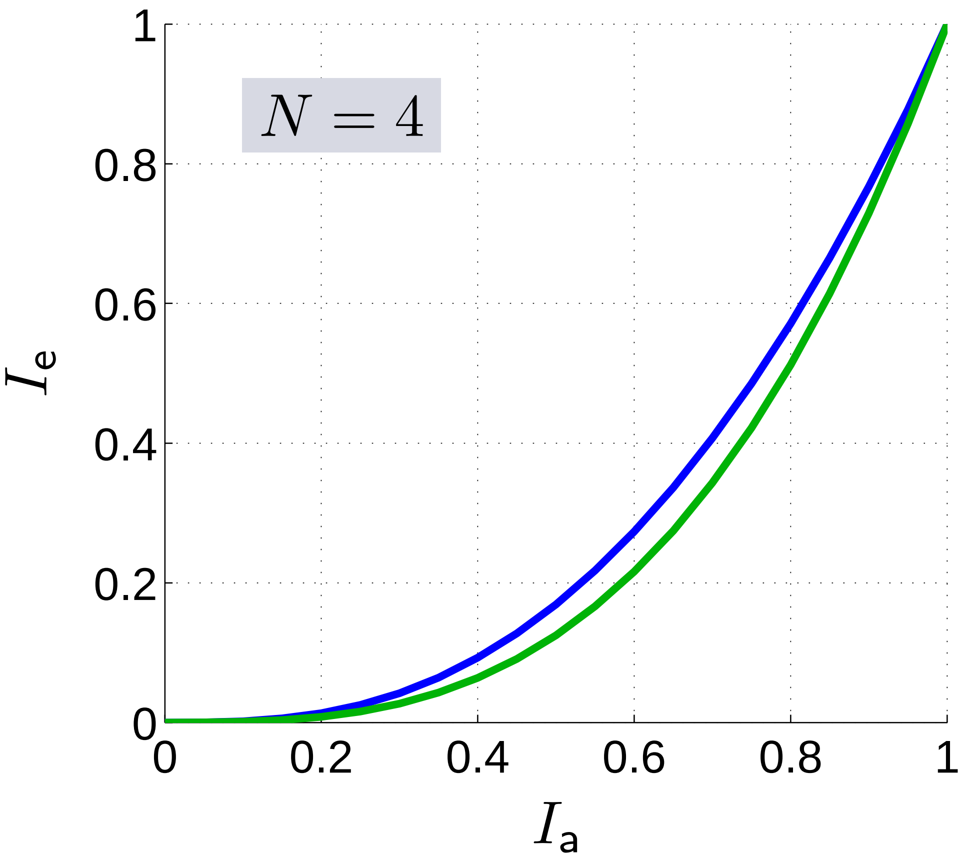

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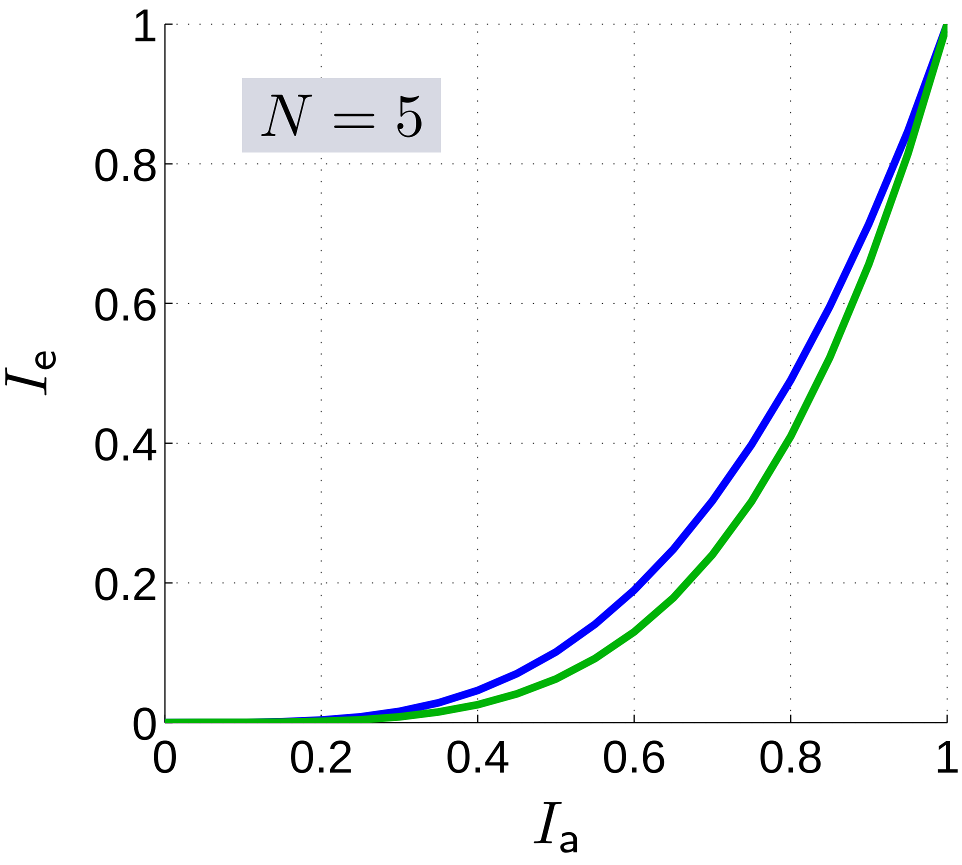

Single Parity-Check Codes: EXIT Functions

- Code length N
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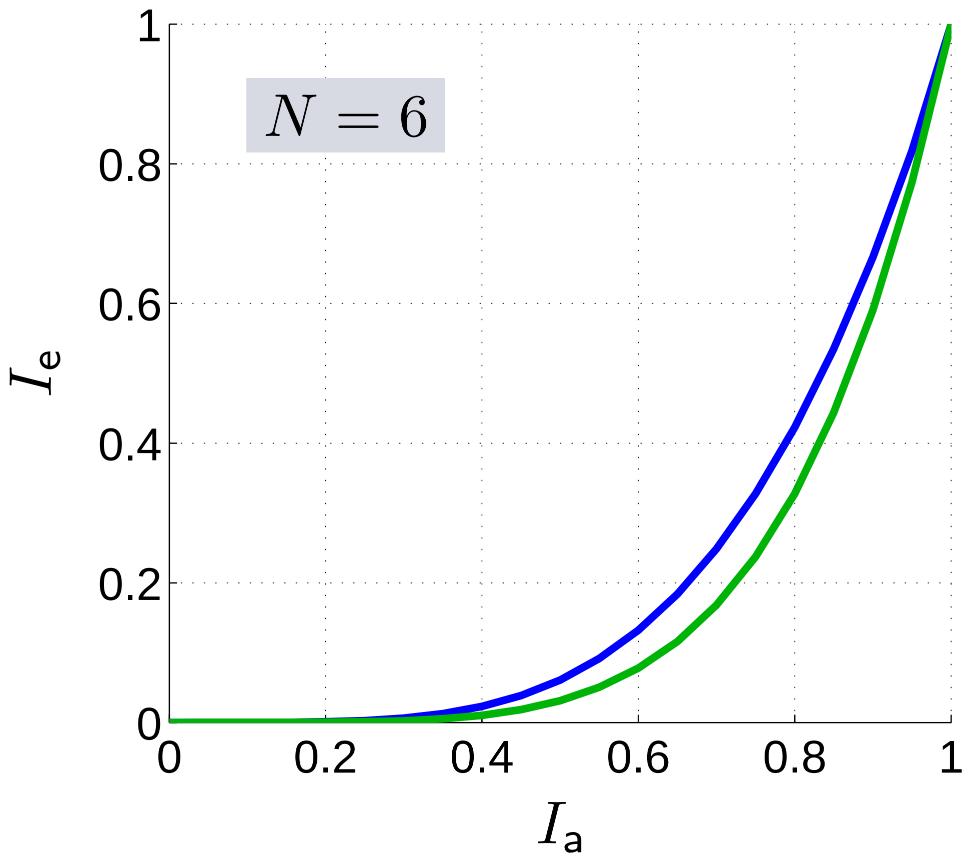

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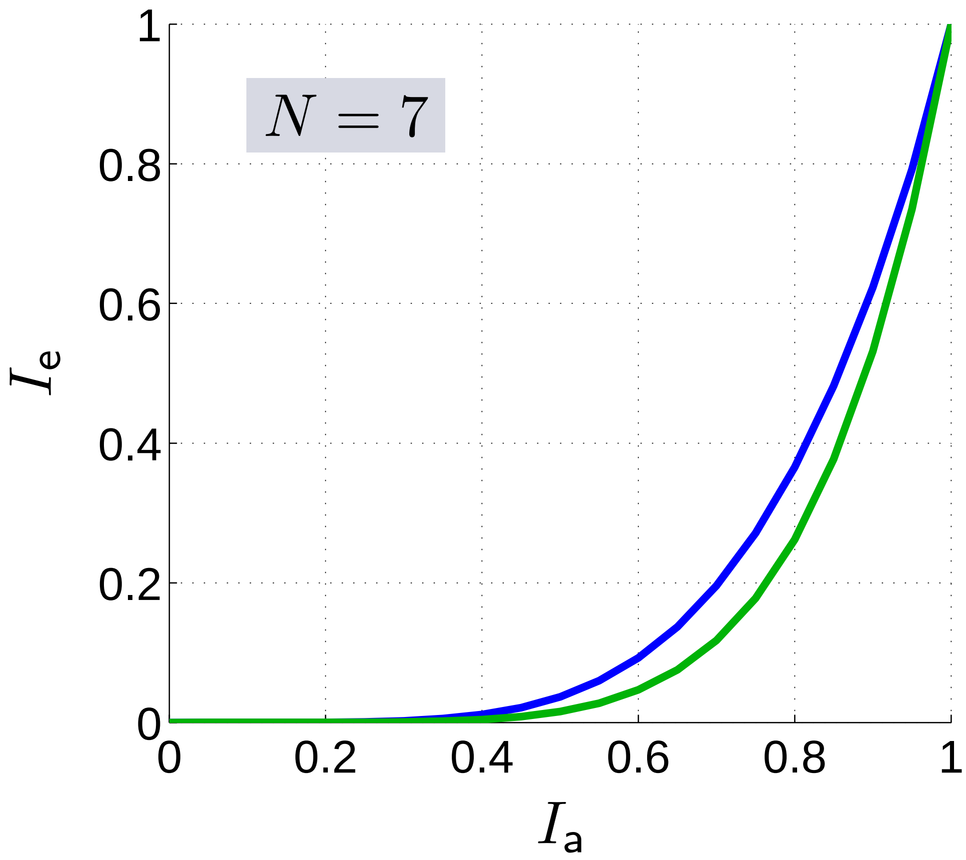

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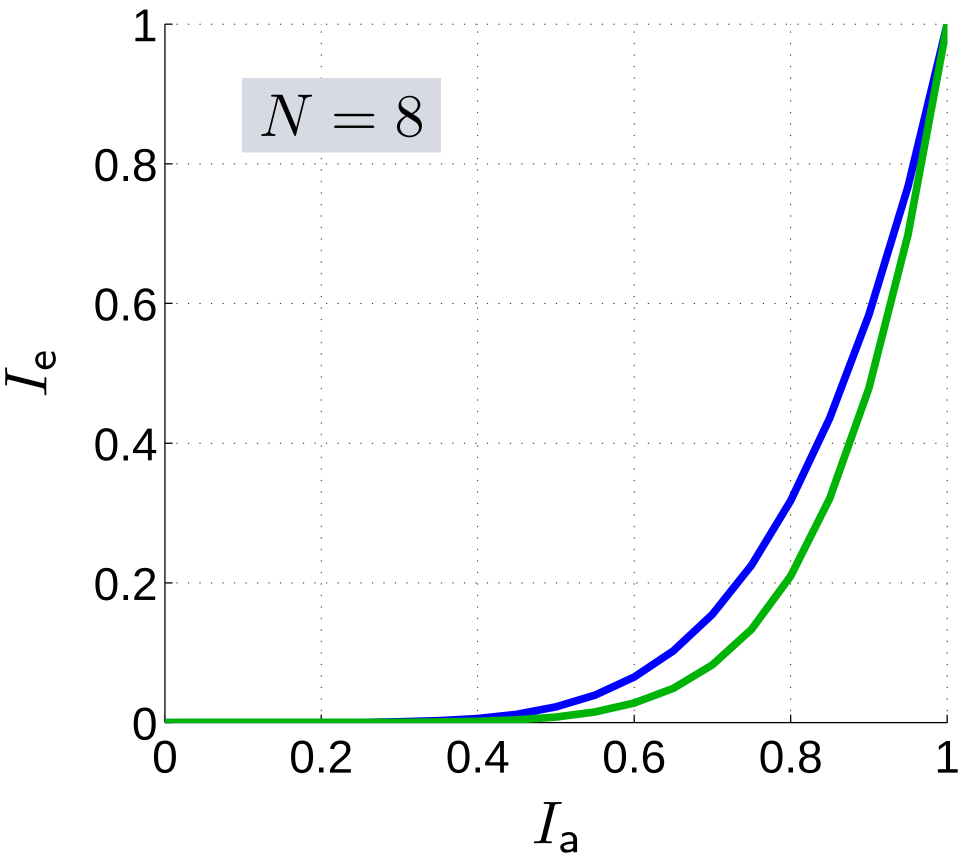

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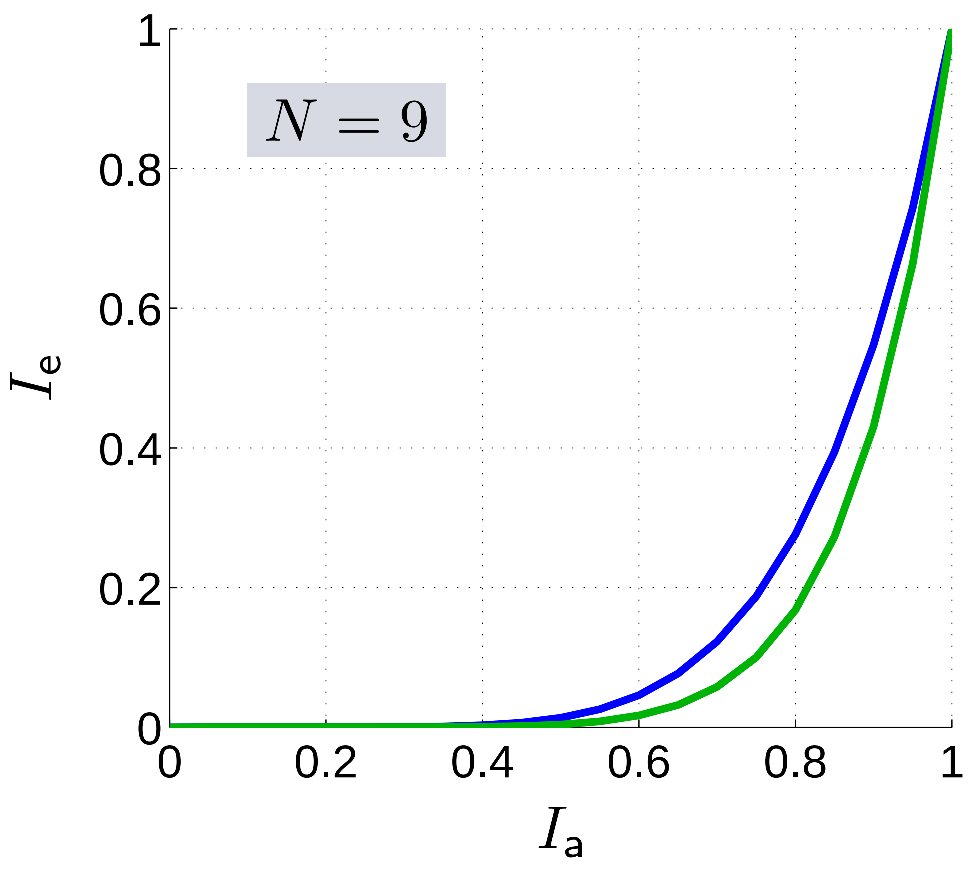
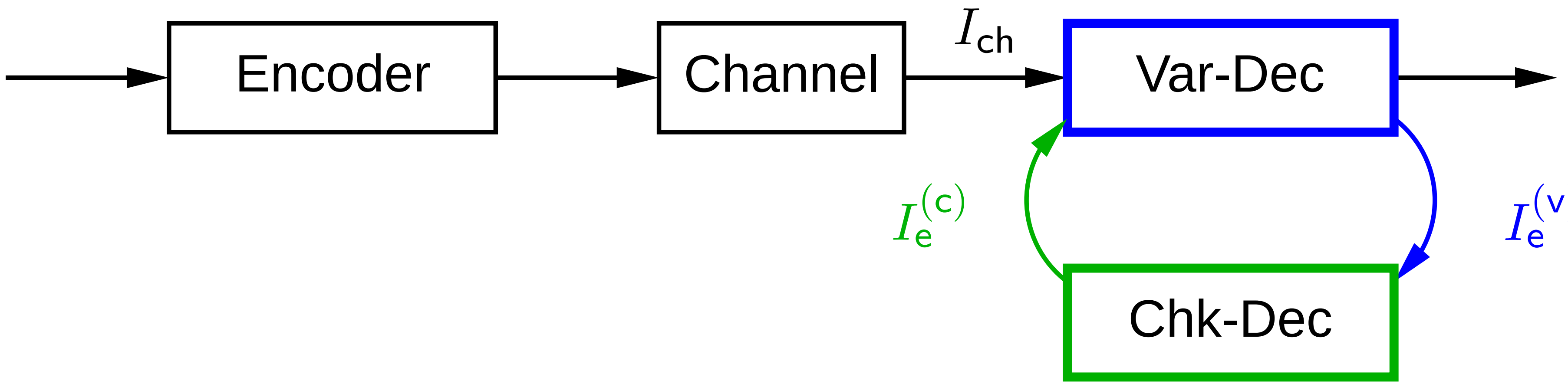


EXIT Functions for LDPC Codes (Reviewed)



- Variable-Node EXIT-Function

$$I_{\text{e}}^{(\text{c})} \mapsto I_{\text{e}}^{(\text{v})} \qquad \text{(Parameter: } I_{\text{ch}}\text{)}$$

- Check-Node EXIT-Function

$$I_{\text{e}}^{(\text{v})} \mapsto I_{\text{e}}^{(\text{c})}$$

- Computation of EXIT functions based on specific model for the virtual extrinsic (a-priori) channel

EXIT-Charts (Reviewed)

- Regular LDPC code

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  $d^{(\mathsf{c})} = 4$
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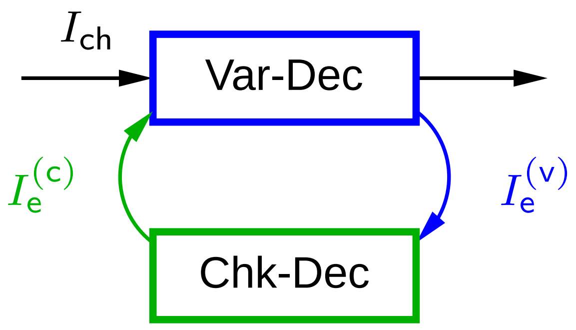
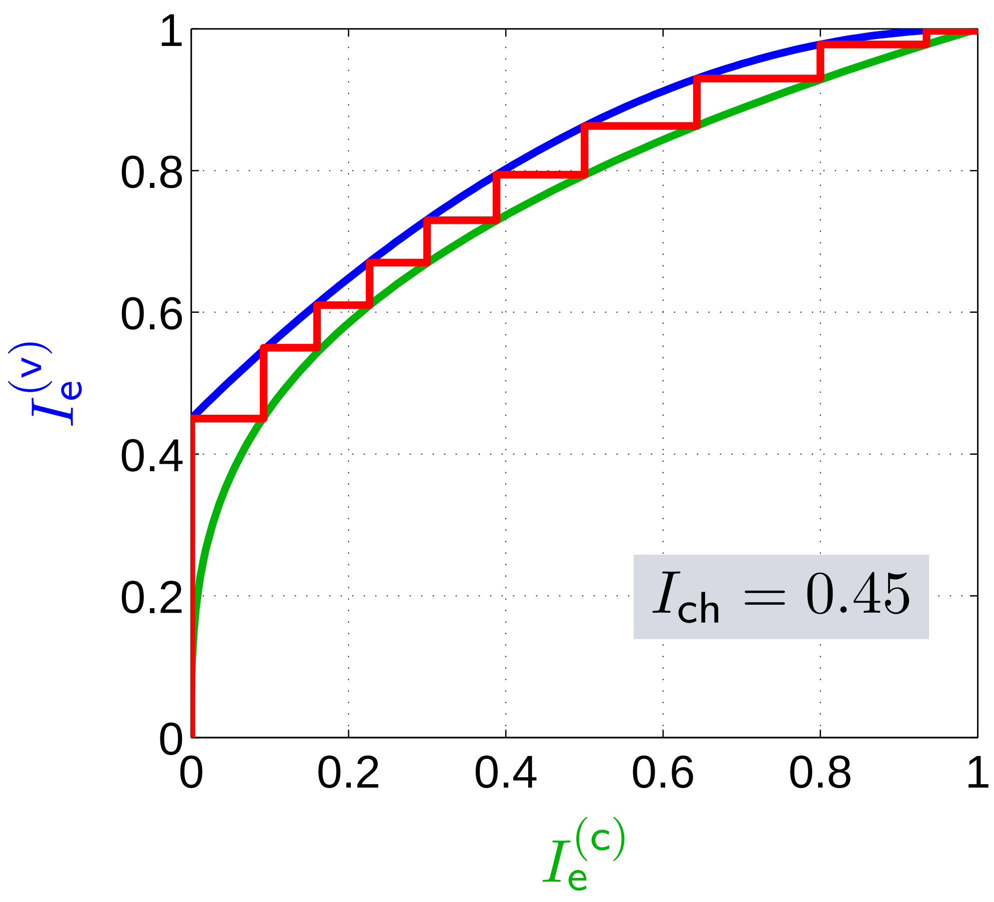
- Iterative decoder:





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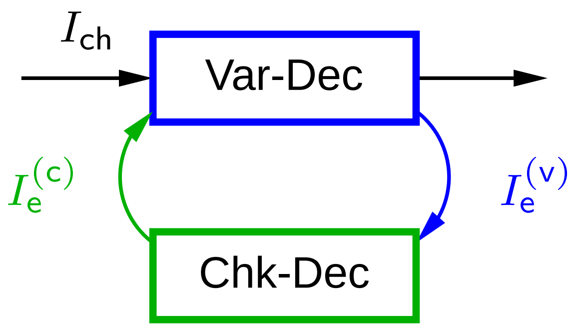

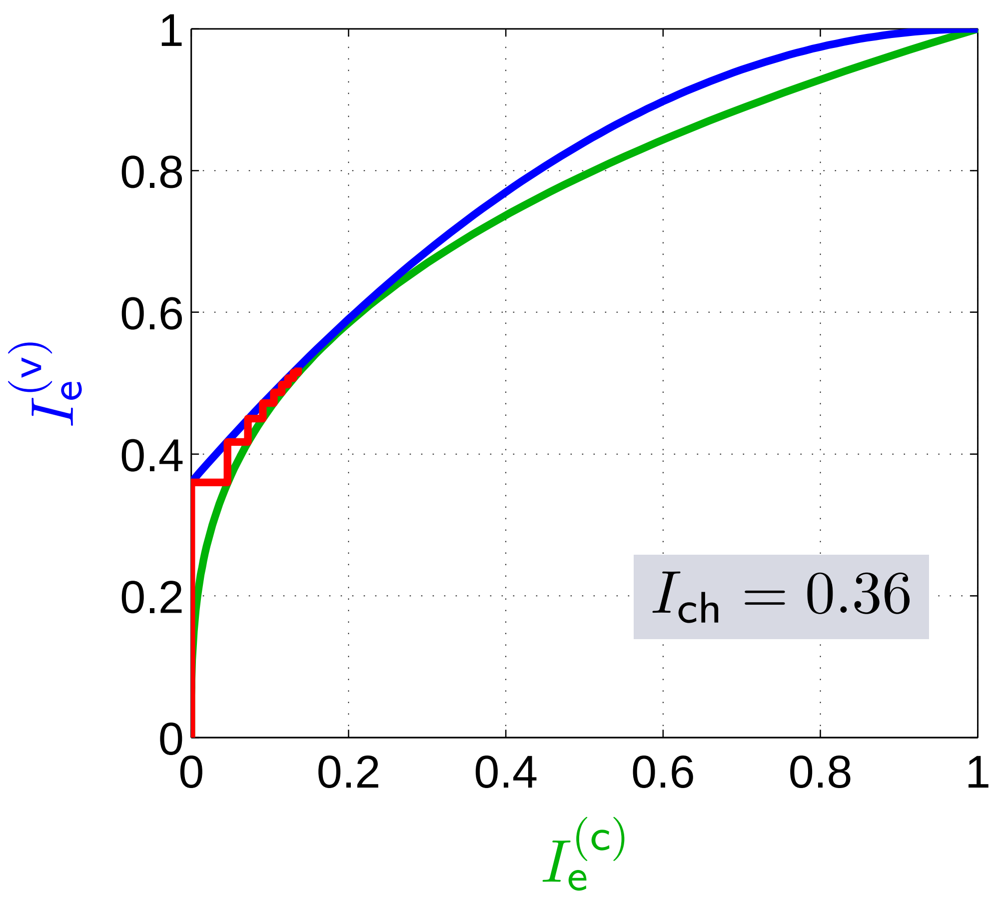

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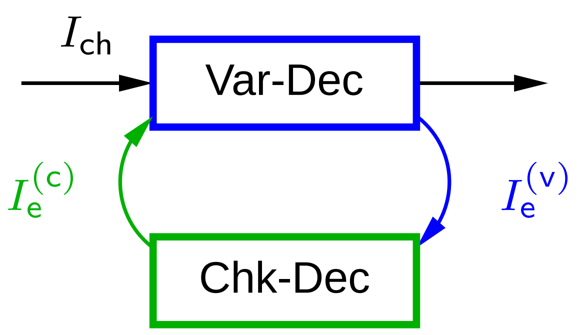

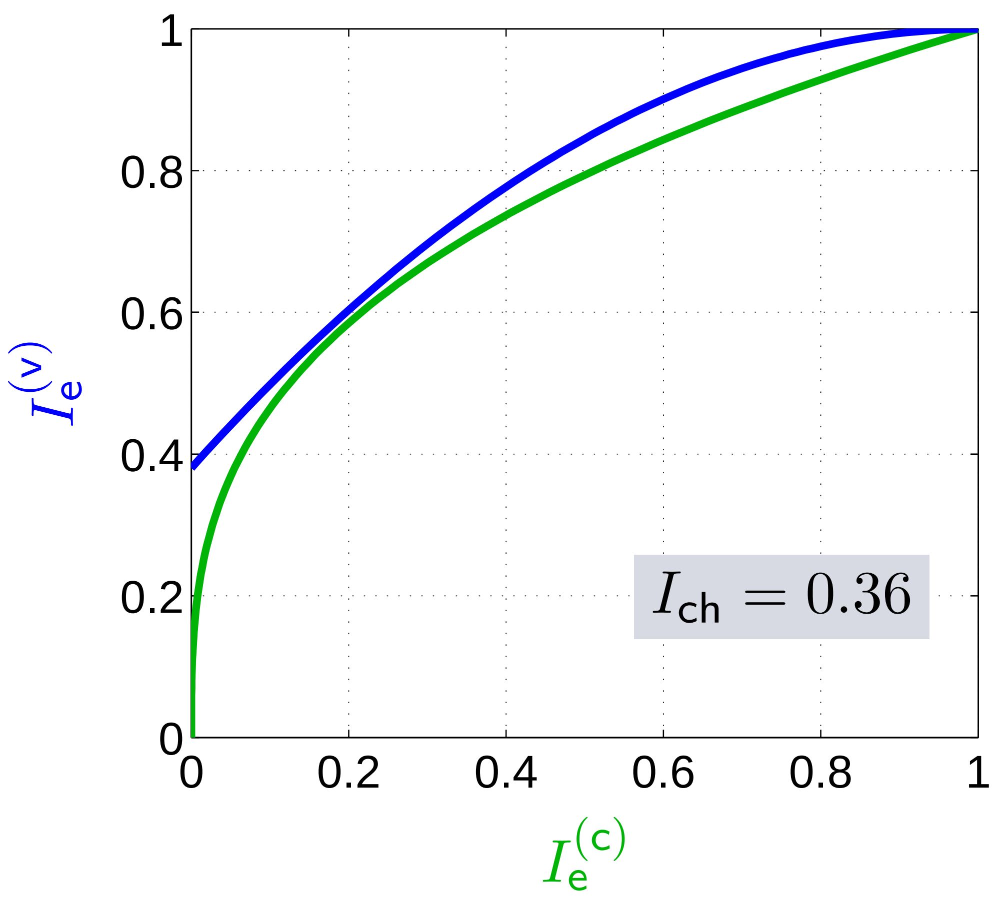

- decoding threshold

Bounds on EXIT Functions

EXIT functions are computed based on a specific model for the virtual extrinsic (a-priori) channels

Disadvantage: Decoding threshold is generally not exact

Idea: Use bounds on information combining

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Variable-node decoding (bounds for repeat codes)

$$f^{\mathsf{par}}_{d^{(\mathsf{v})}}(I_{\mathsf{ch}}, I_{\mathsf{e}}^{(\mathsf{c})}, \ldots, I_{\mathsf{e}}^{(\mathsf{c})}) \;\leq\; I_{\mathsf{e}}^{(\mathsf{v})} \;\leq\; 1 - (1 - I_{\mathsf{ch}})(1 - I_{\mathsf{e}}^{(\mathsf{c})})^{d^{(\mathsf{v})}-1}$$

Check-node decoding (bounds for SPC codes)

$$(I_{\mathsf{e}}^{(\mathsf{v})})^{d^{(\mathsf{c})}-1} \;\leq\; I_{\mathsf{e}}^{(\mathsf{c})} \;\leq\; f^{\mathsf{ser}}_{d^{(\mathsf{c})}-1}(I_{\mathsf{e}}^{(\mathsf{v})}, \ldots, I_{\mathsf{e}}^{(\mathsf{v})})$$

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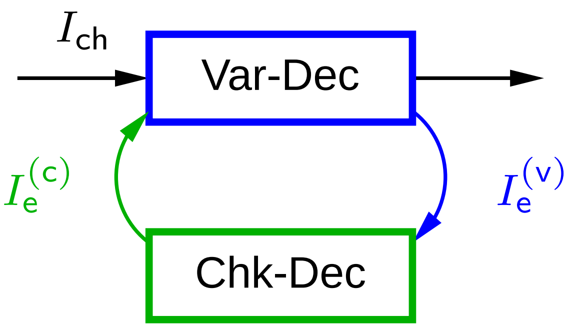

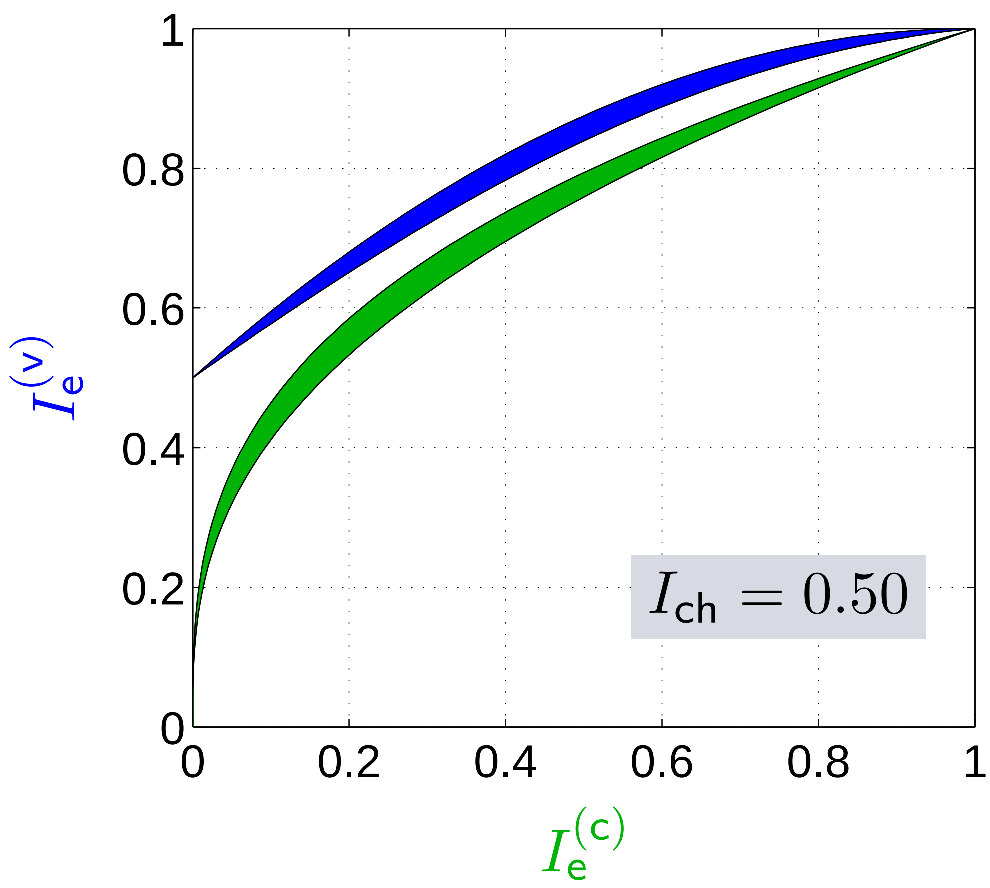

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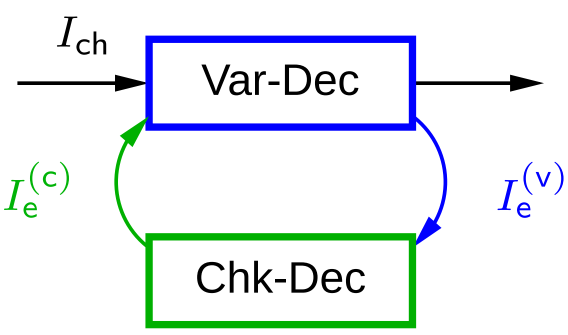

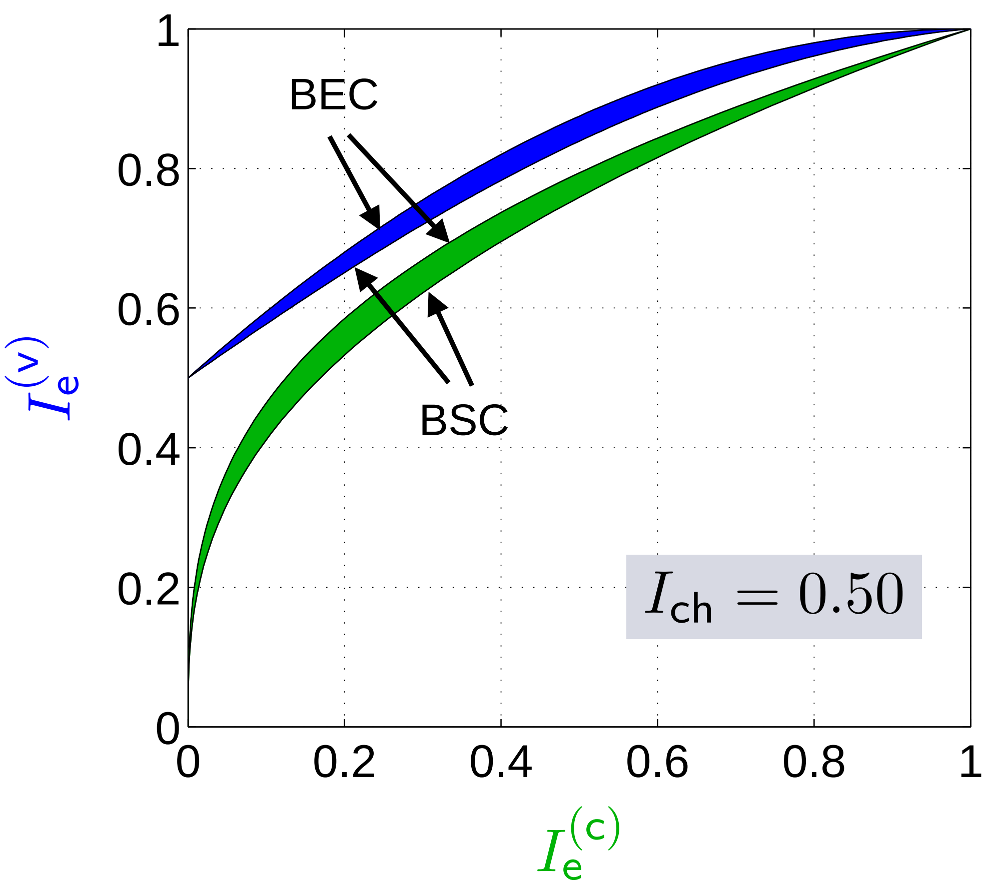

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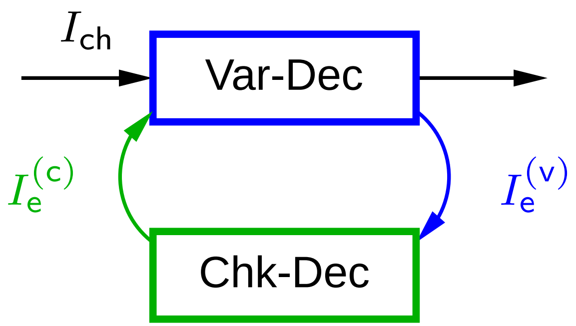

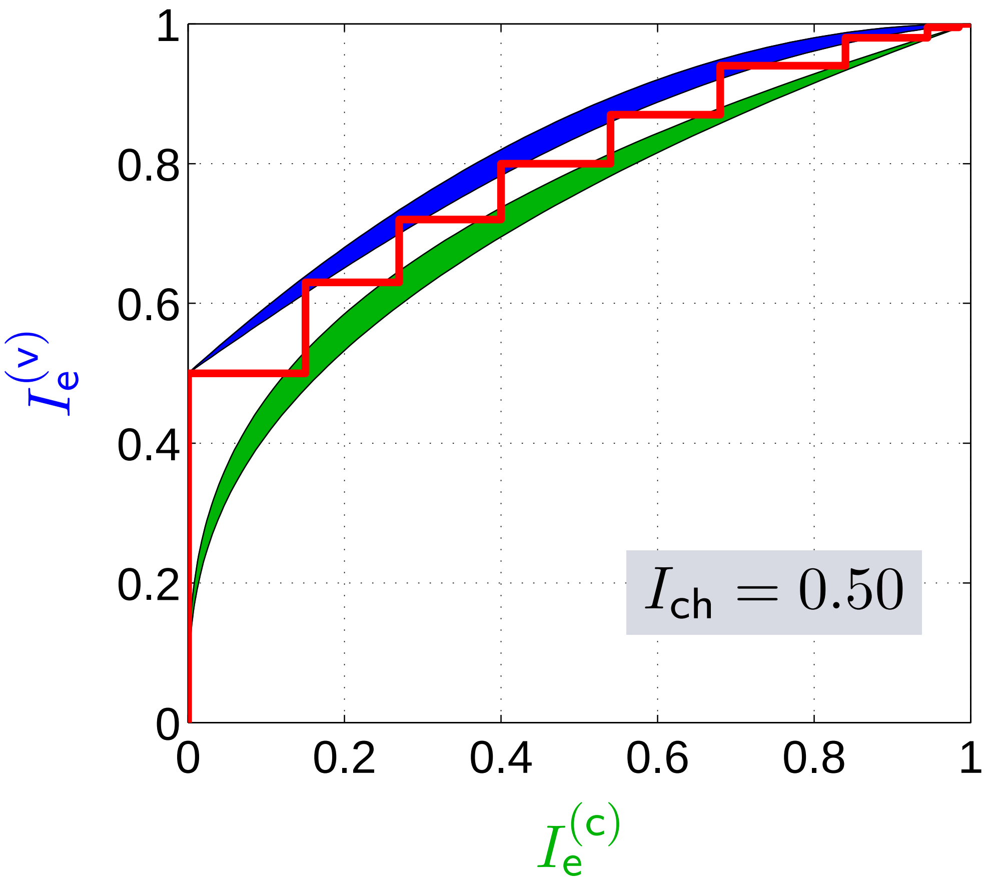

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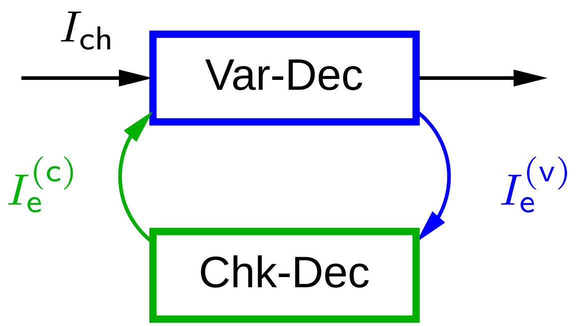

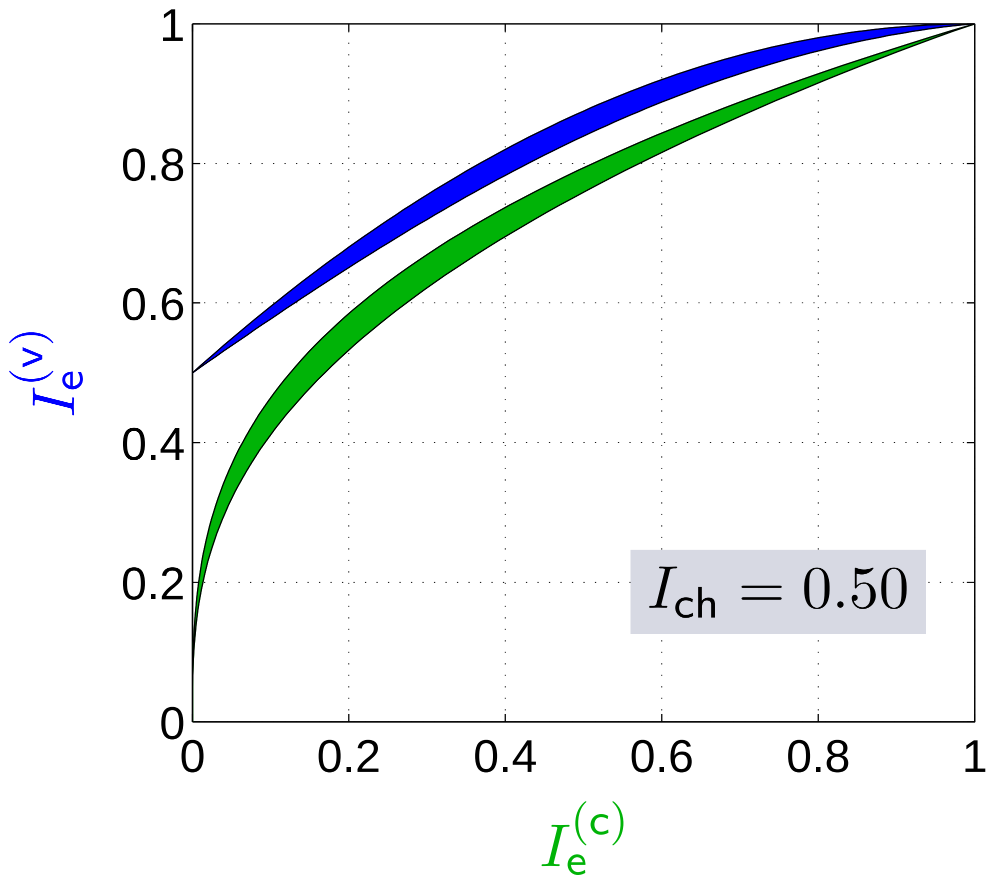

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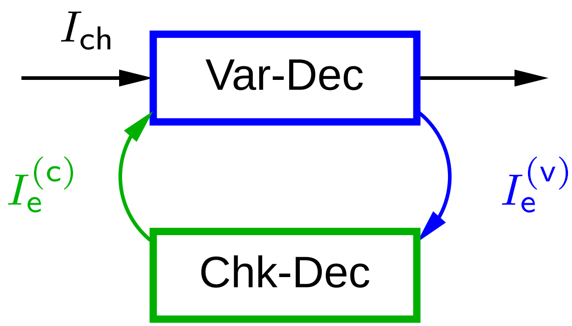

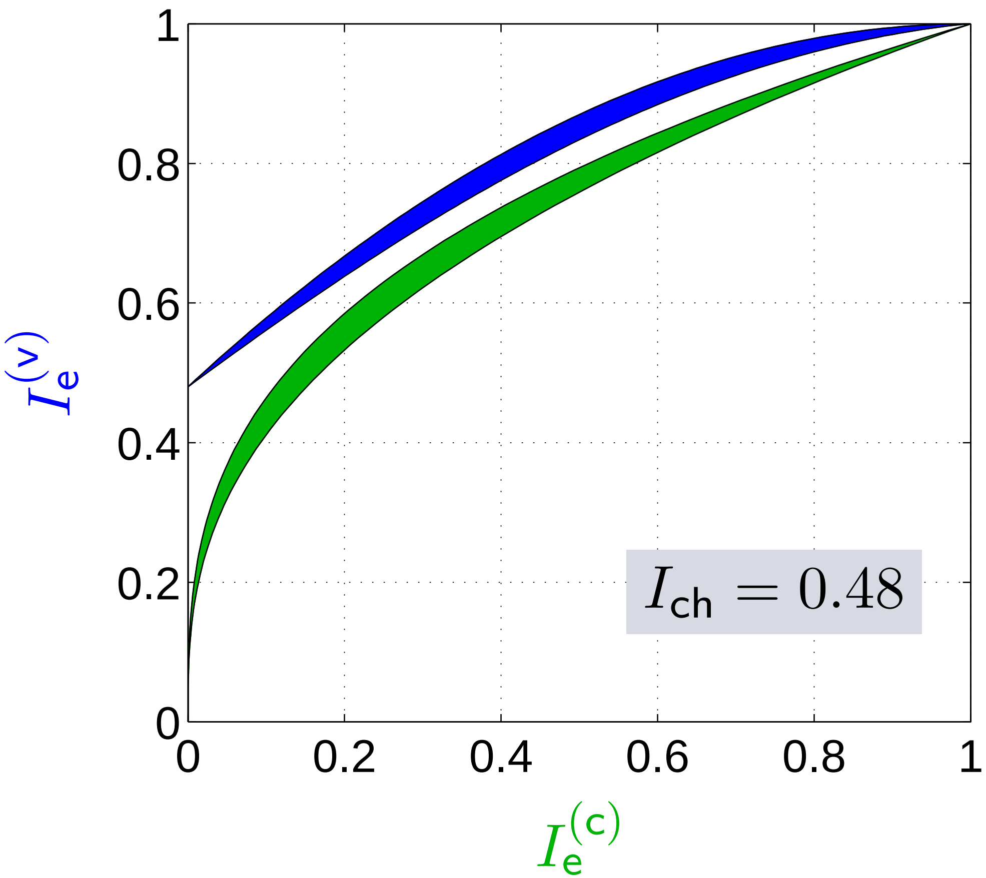

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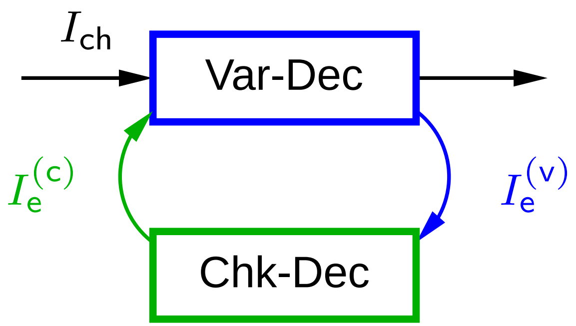

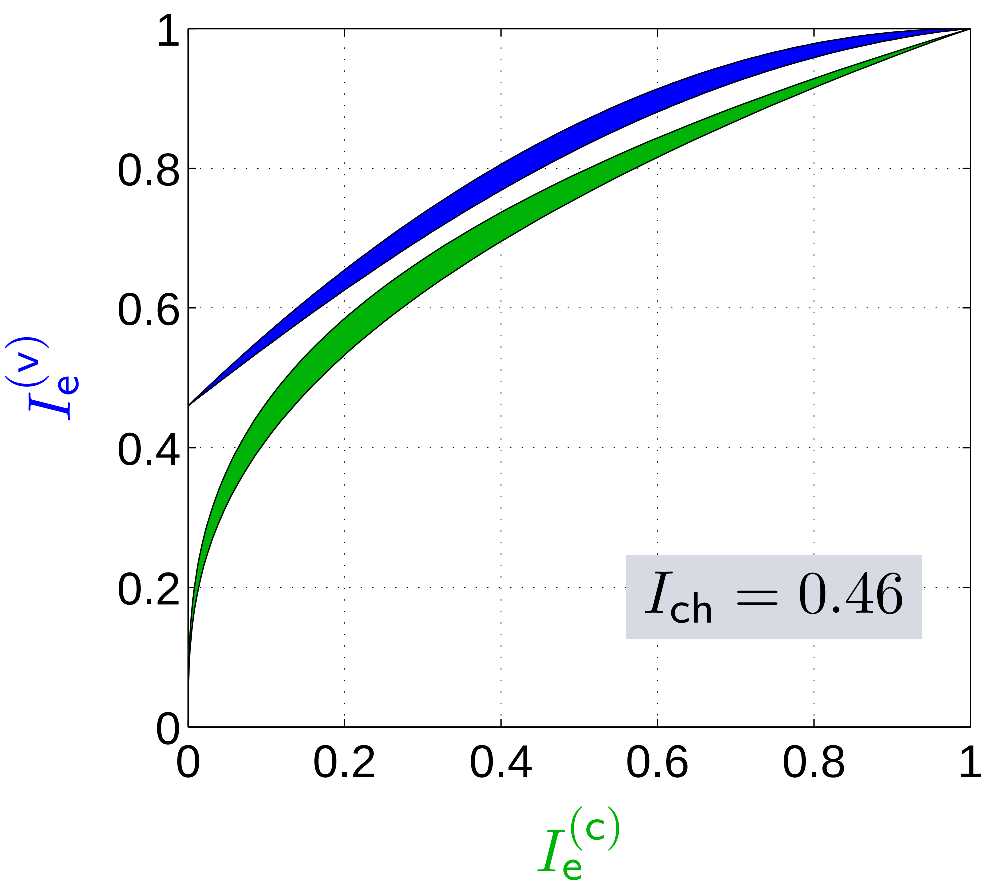

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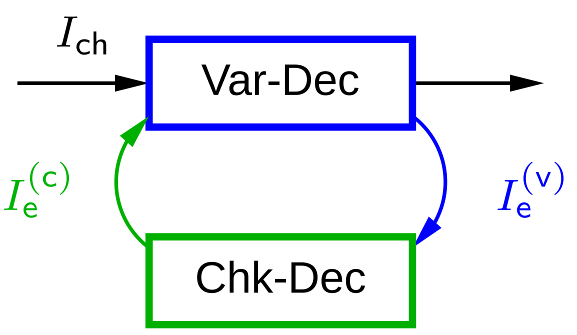

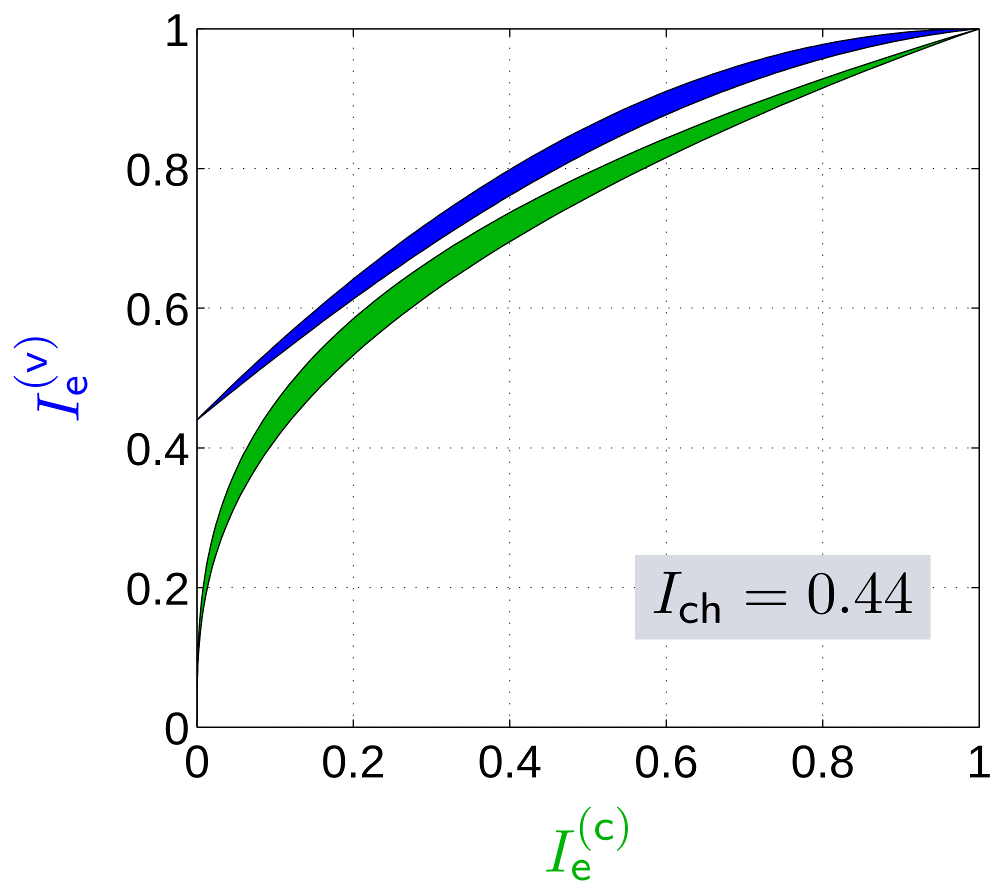

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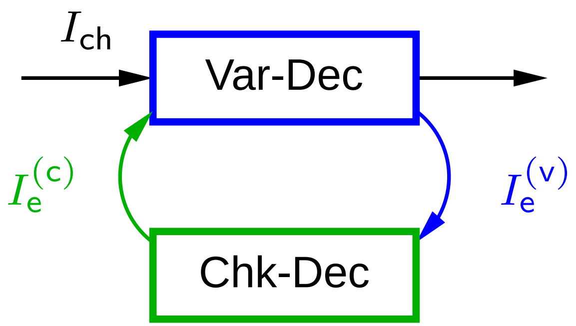

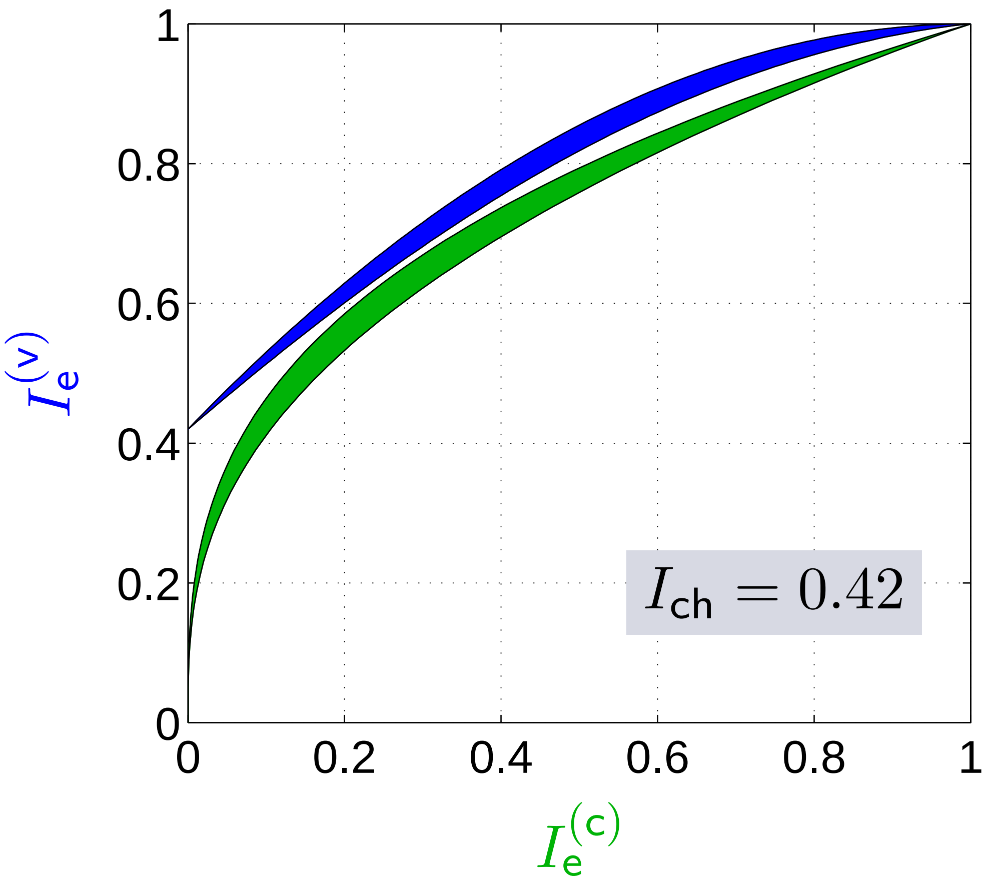

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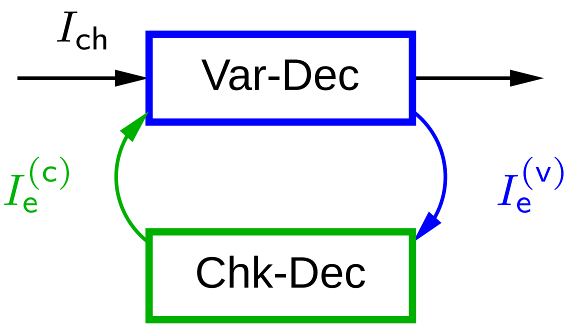

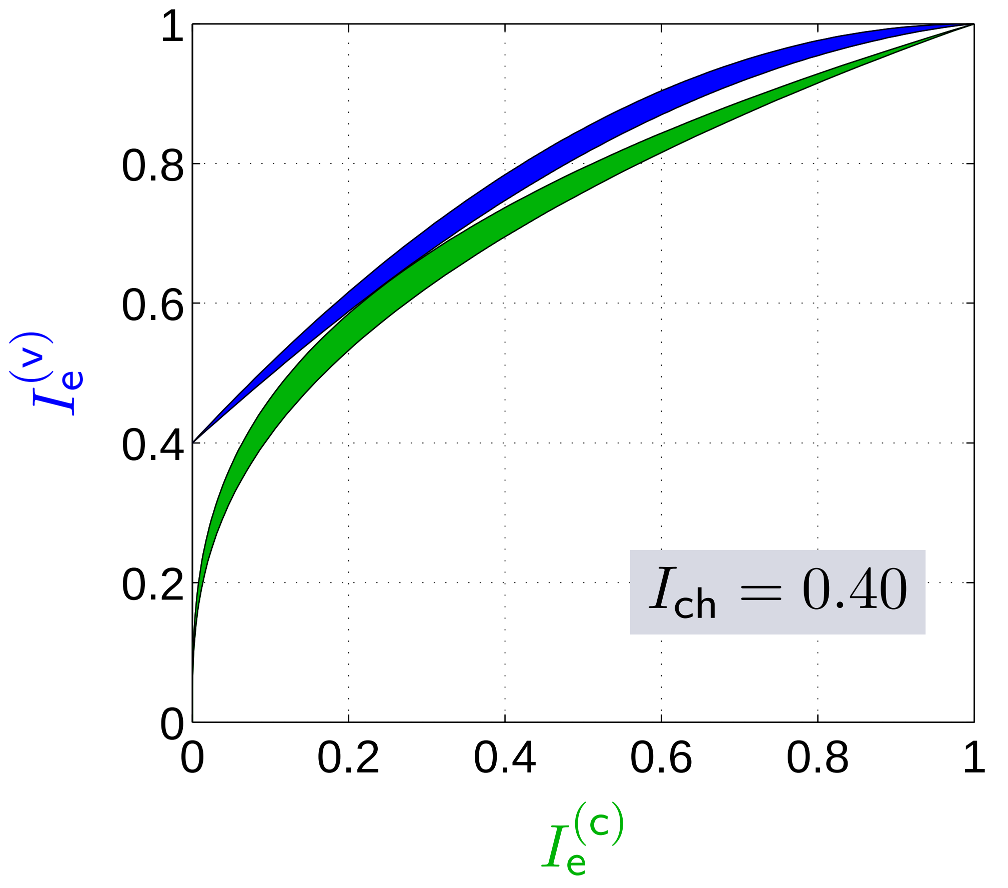

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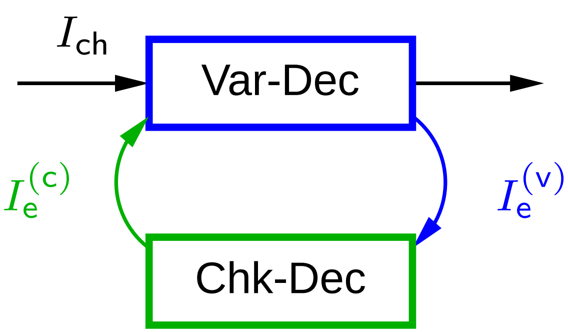

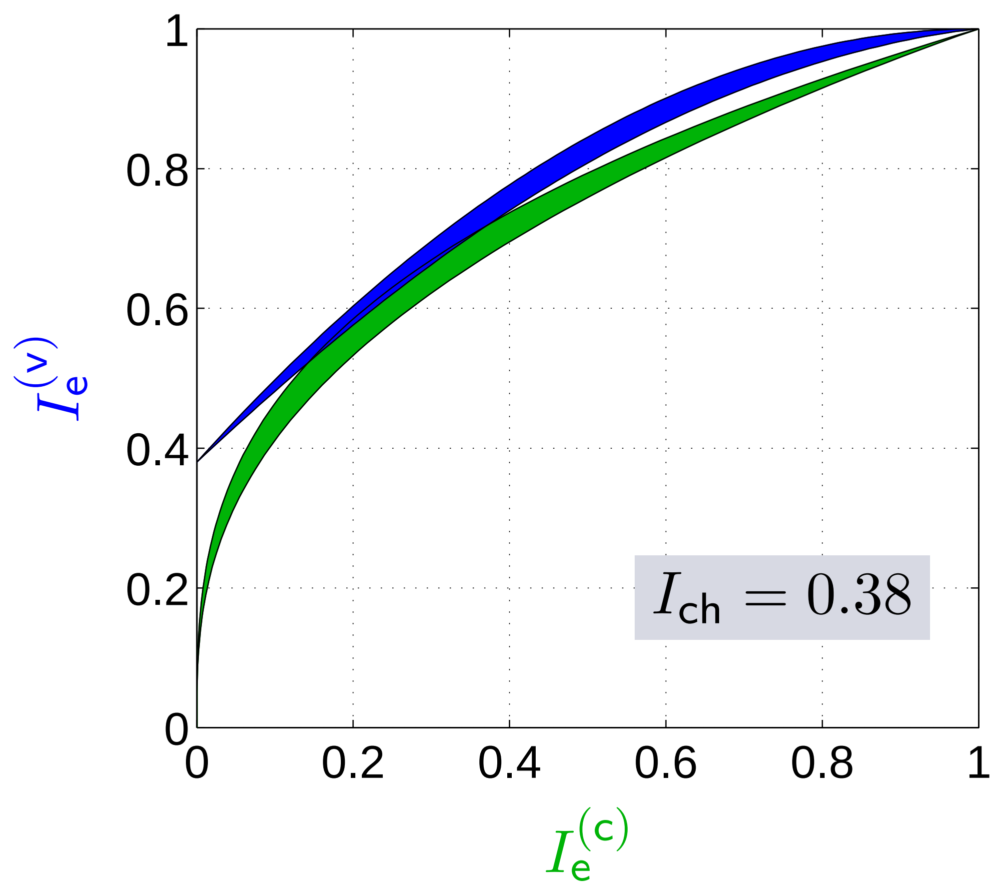

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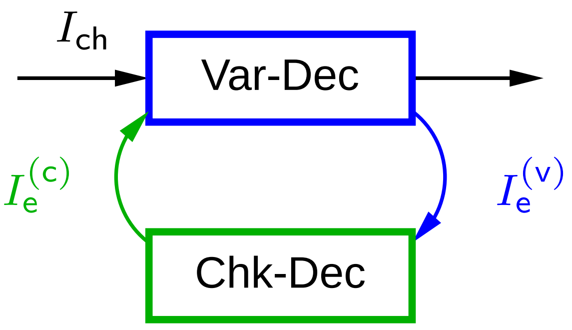

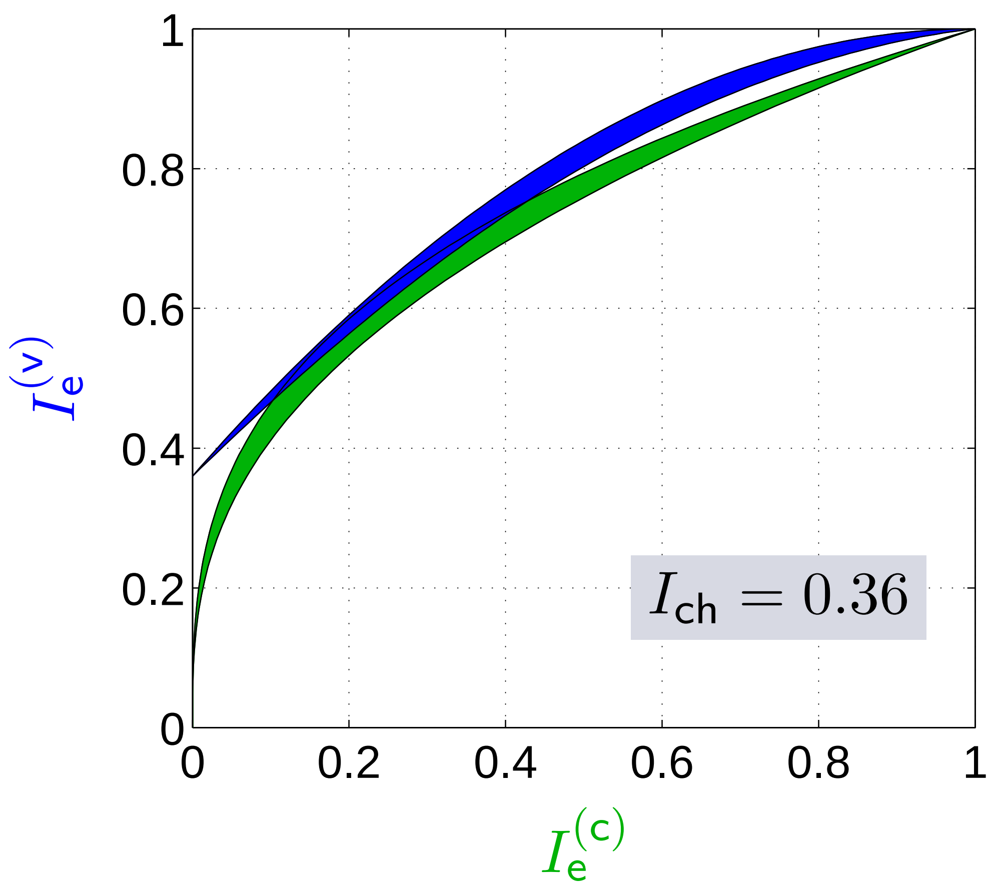

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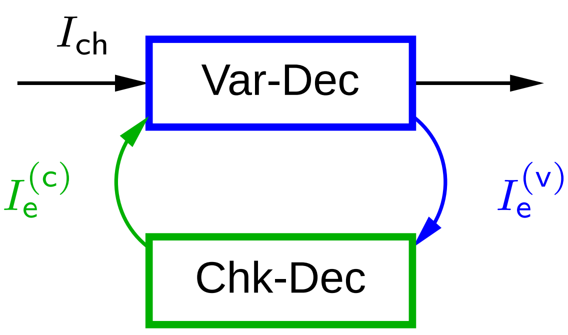

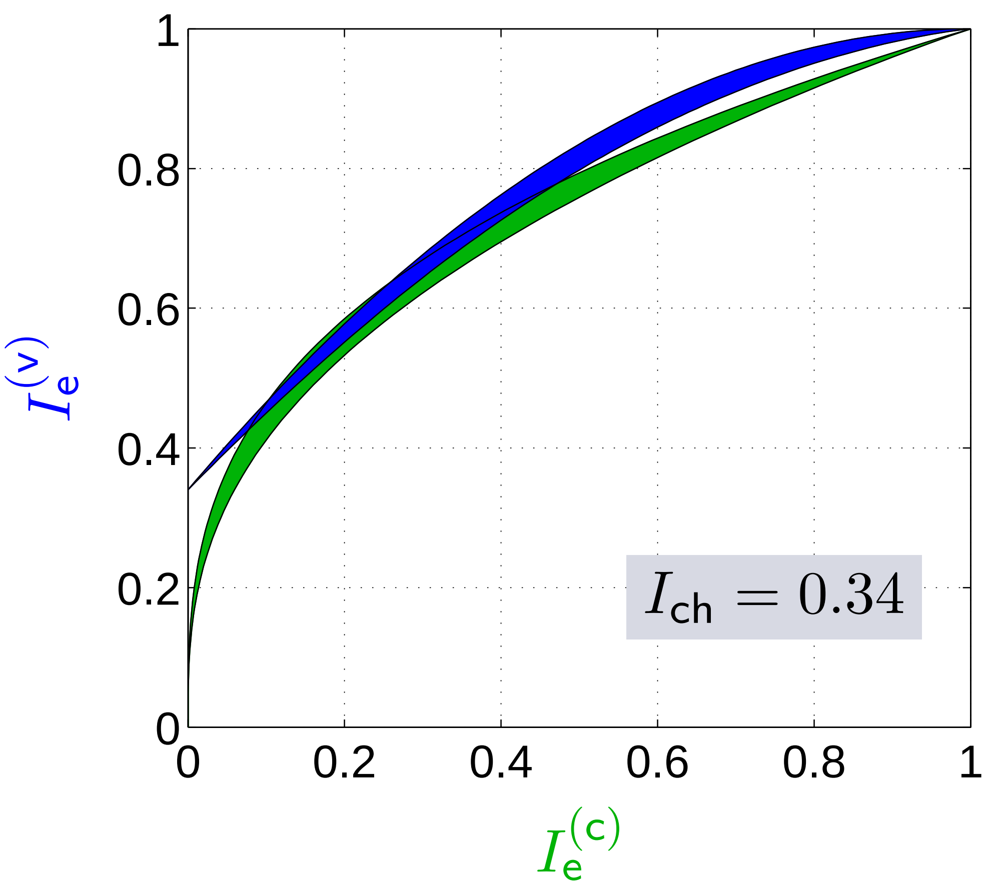

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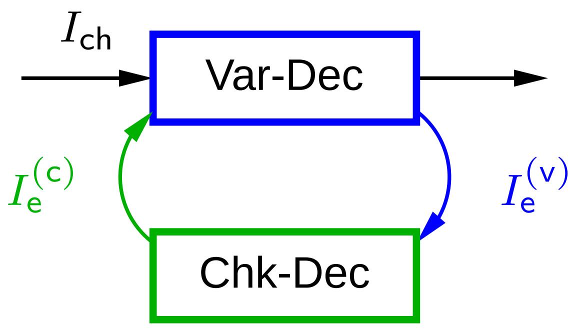

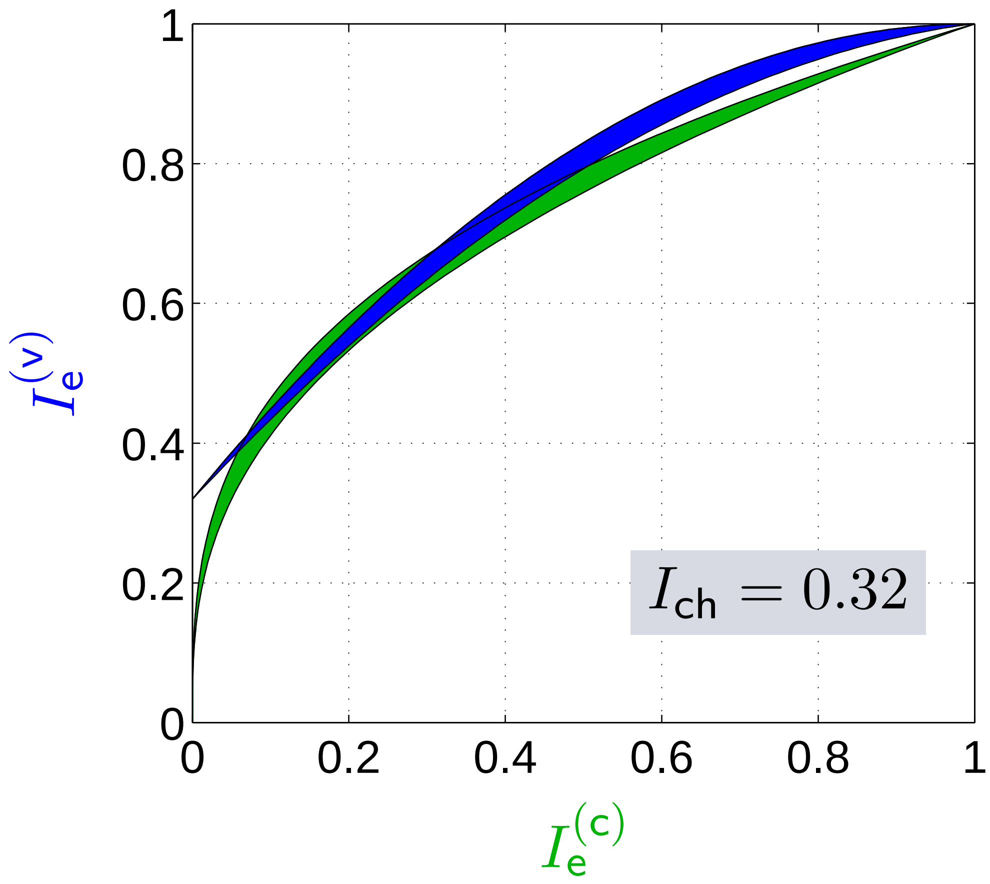

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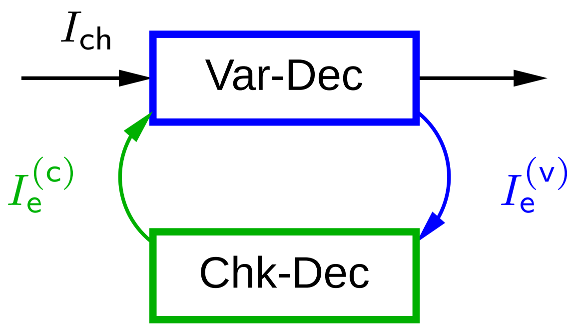

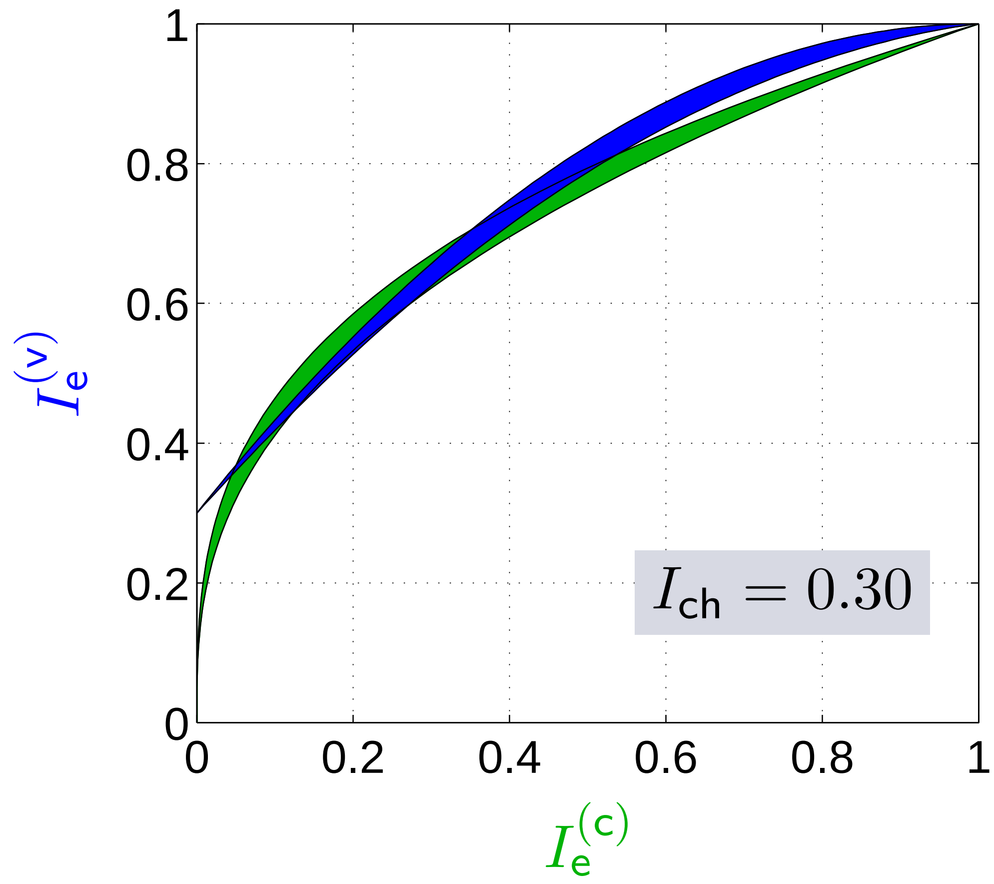

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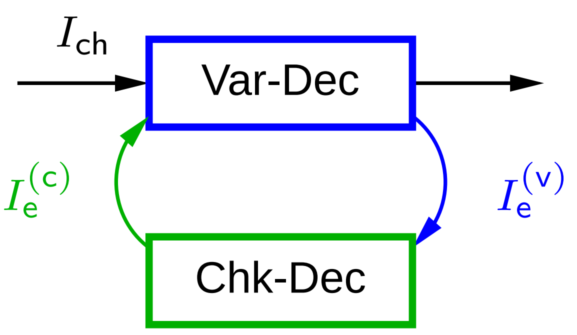

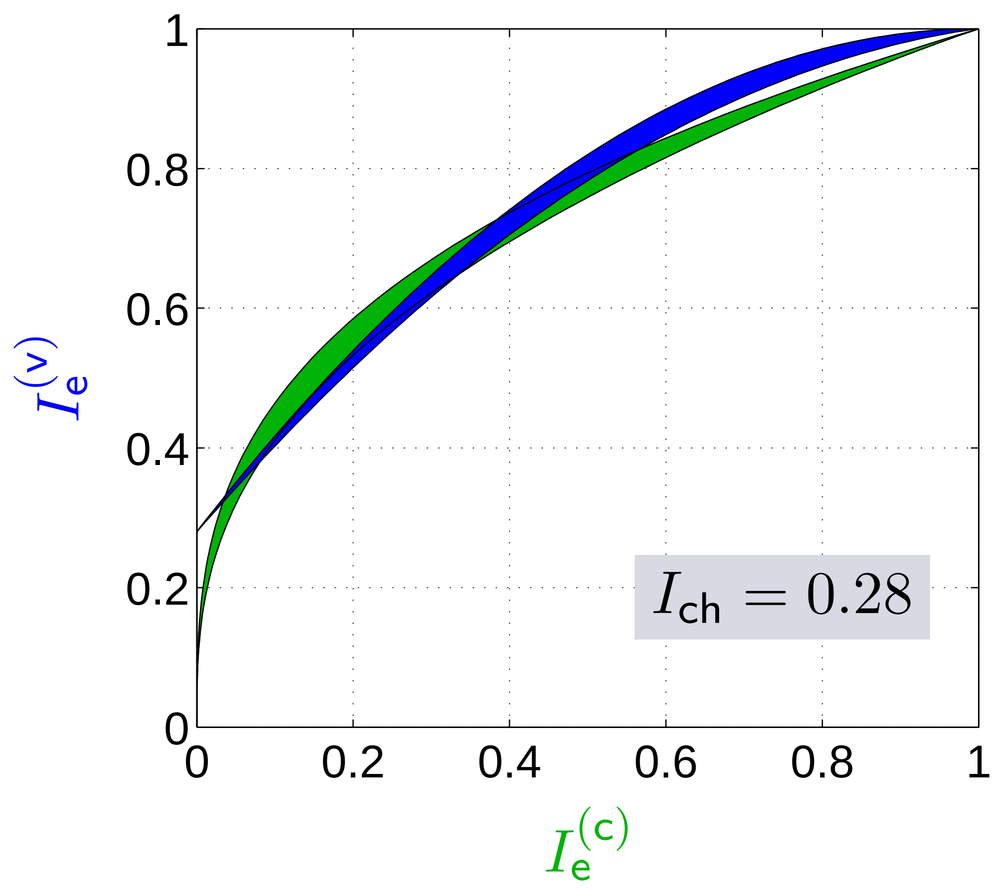

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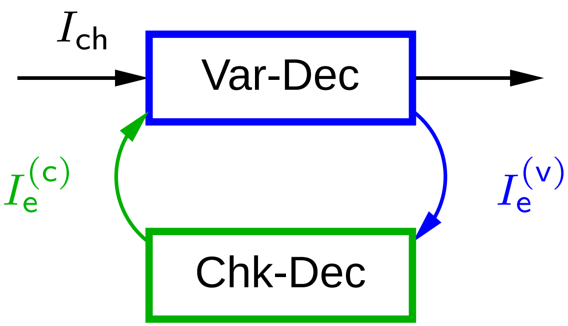

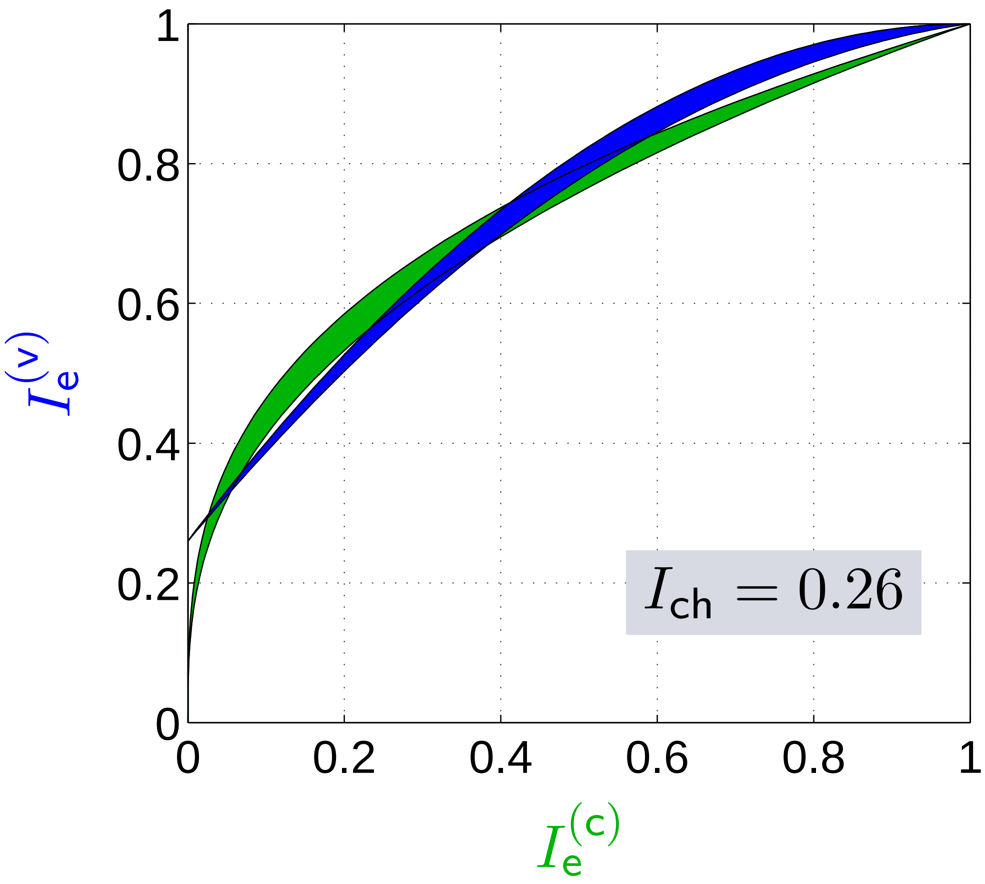

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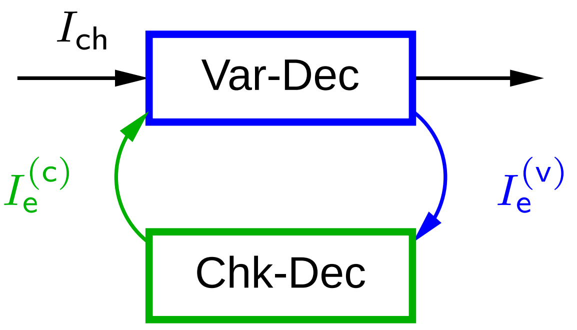

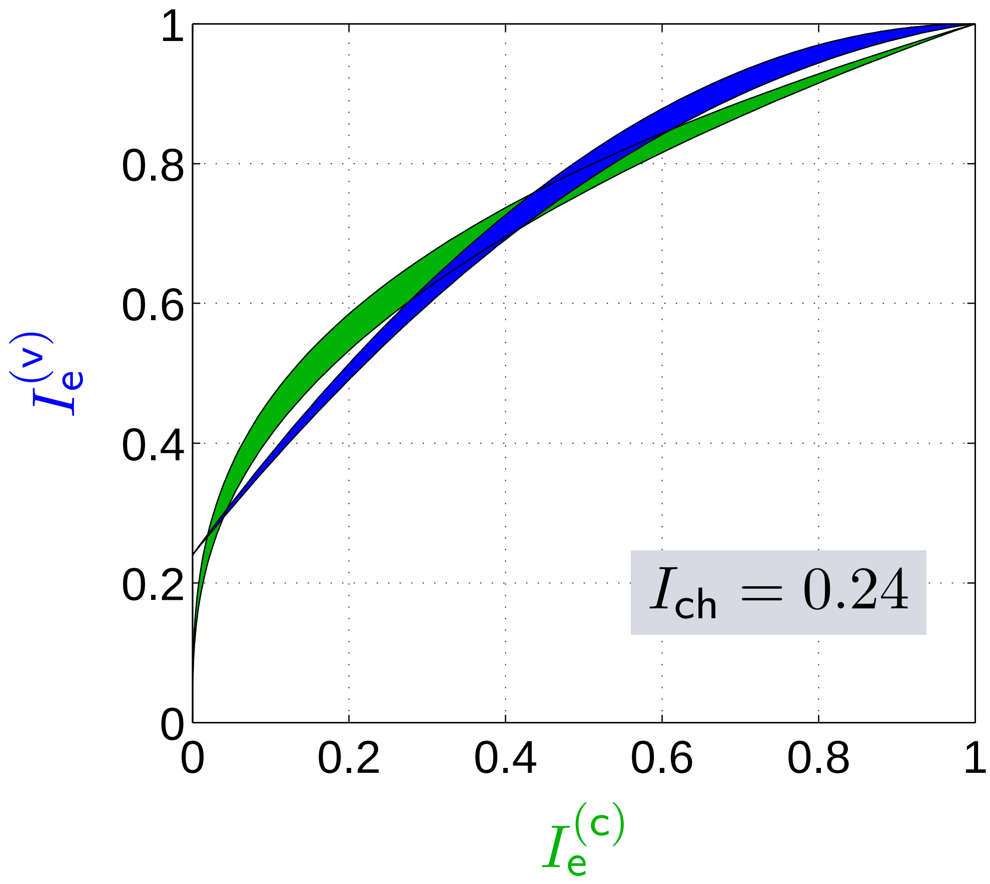

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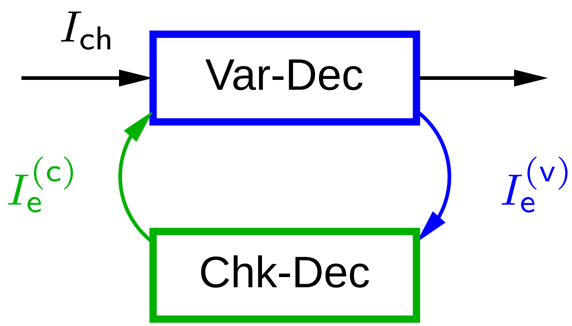

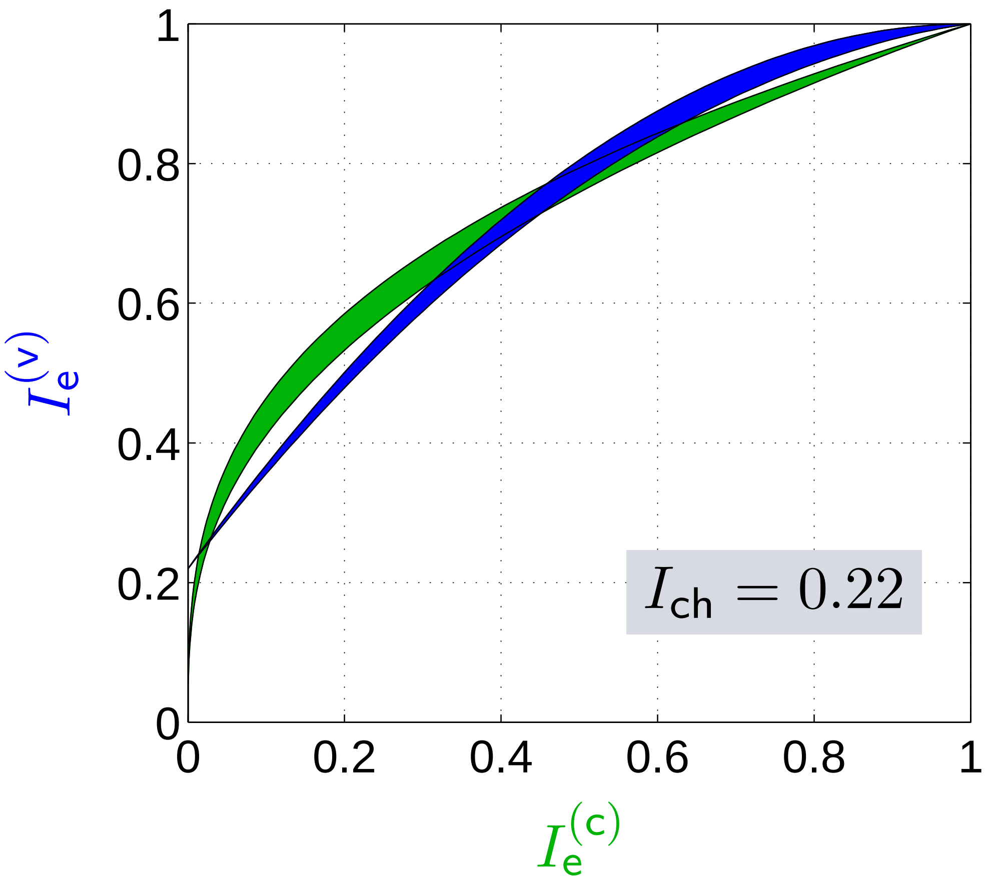

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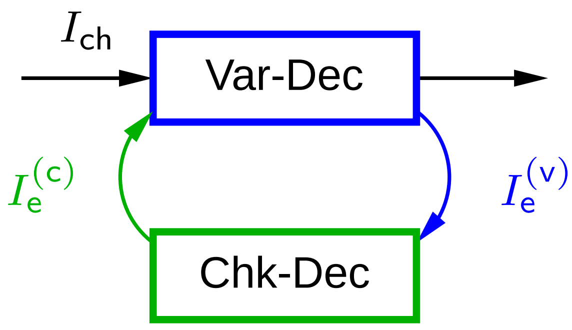

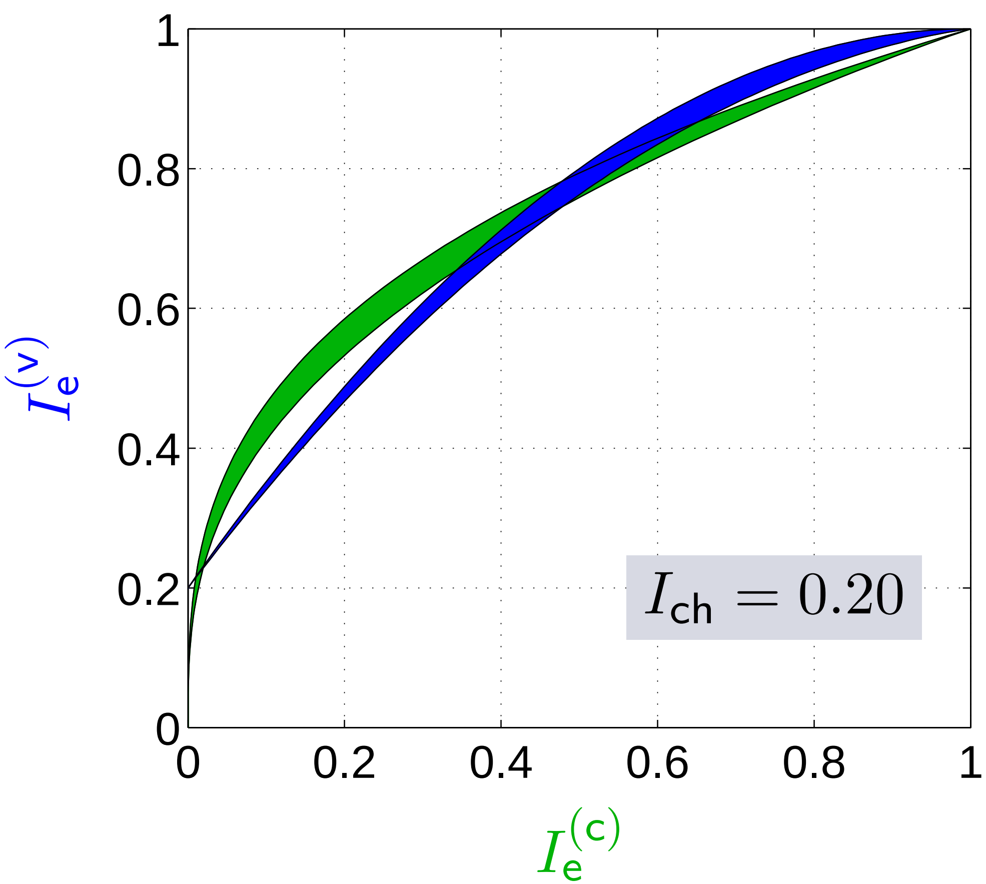

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Bounds on Decoding Threshold

- Decoding thresholds analytically determined
- Depend only on the mutual information (capacity) of the communication channel
- Valid for all(!) BISMCs

Advantage:

More general than the conventional EXIT-Chart Method

Disadvantage:

“Only” bounds on the threshold

Literature

[1] I. Land, S. Huettinger, P. A. Hoeher, and J. Huber, “Bounds on information combining,” *IEEE Trans. Inform. Theory*, vol. 51, no. 2, pp. 612–619, Feb. 2005.

[2] ——, “Bounds on mutual information for simple codes using information combining,” *Ann. Télécommun.*, vol. 60, no. 1/2, pp. 184–214, Jan./Feb. 2005.

[3] I. Sutskover, S. Shamai (Shitz), and J. Ziv, “Extremes of information combining,” *IEEE Trans. Inform. Theory*, vol. 51, no. 4, pp. 1313–1325, Apr. 2005.

[4] I. Land, “Reliability information in channel decoding – practical aspects and information theoretical bounds,” Ph.D. dissertation, University of Kiel, Germany, 2005.