Forschungszentrum Telekommunikation Wien

Theory and Design of Turbo and Related Codes

Lecture 13

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http://userver.ftw.at/~jossy/turbo/index.html

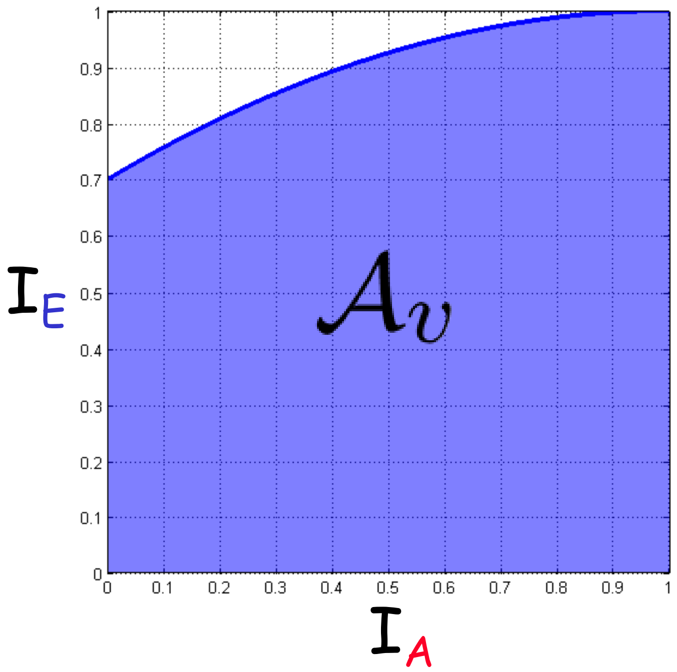
Kplus
Kompetenzzentren-Programm

EXIT Charts: what we did so far...

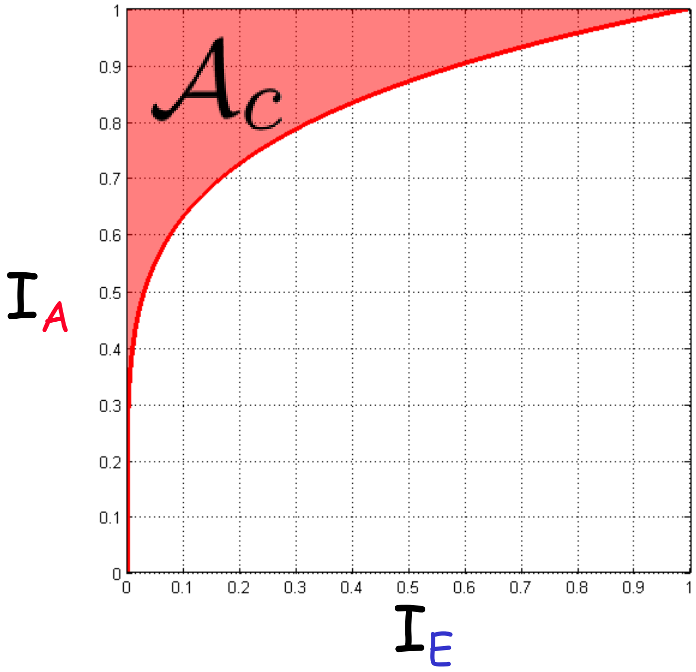
- Extrinsic Channel Model
- EXIT Chart of Regular LDPC Codes
- EXIT Charts for Mixtures / Irregular LDPC Codes
- Area Property
- Code Design via Curve Fitting

Area of LDPC Component Codes

$$\mathcal{A}_v = 1 - \frac{1 - C}{d_v}$$



$$\mathcal{A}_c = \frac{1}{d_c}$$



Area and Rate Loss

$$R = 1 - \frac{d_v}{d_c} = 1 - \frac{1-C}{\gamma} = \frac{C-(1-\gamma)}{\gamma} < C$$

If $\gamma \rightarrow 1$ we can transmit at rates that approach capacity.
If $\gamma < 1$ we are bounded from capacity.

$\gamma \rightarrow 1$ means that $1 - A_v = A_c$

Furthermore, the curves must not intersect.

$\Rightarrow$ The curves have to be matched.

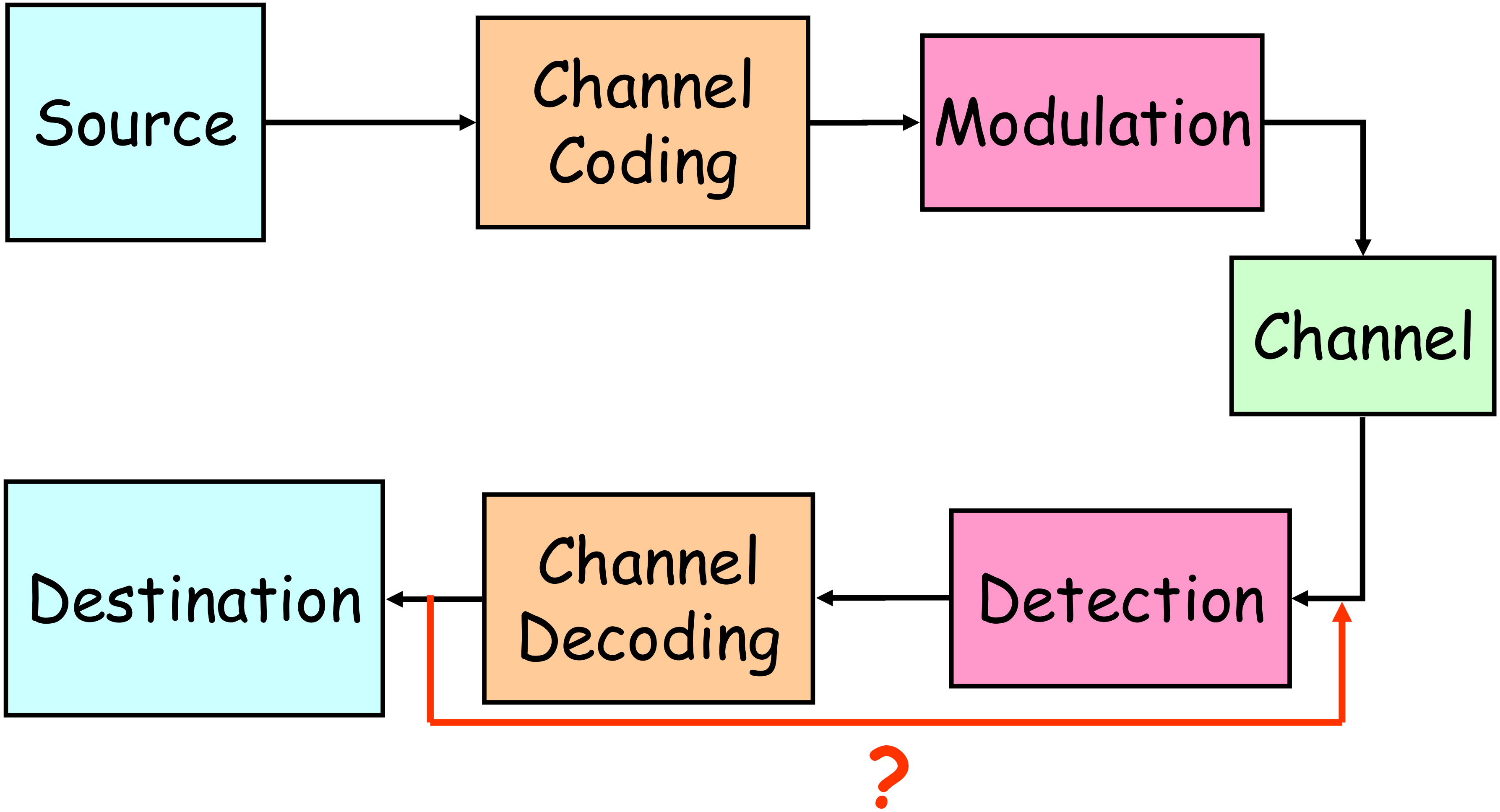
Code design reduces to curve fitting!

Area Property

- Area Property is exact only for the Binary Erasure Channel

- Nonetheless, we use curve fitting on EXIT charts to design codes for other channels with success

Code Design for Modulation and Detection



Source

Channel Coding

Modulation

Channel

Detection

Channel Decoding

Destination

?

Code Design for Modulation and Detection

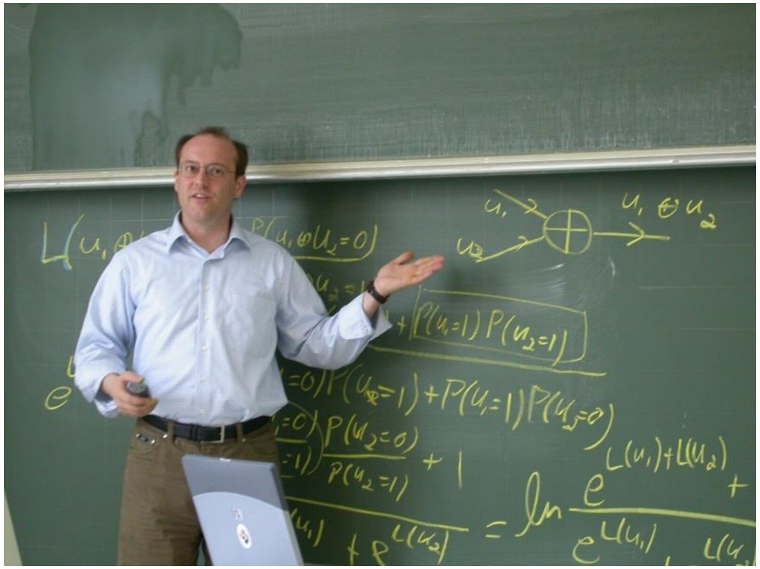
- No iteration over the detector
  ➔ design code optimally for the "channel"
  !!! detection loss cannot be compensated for…

- With iteration over the detector
  ➔ the detector is part of the iterative decoder
  !!! the code must be designed for the detector

Code Design for Modulation and Detection

"Design of Low-Density Parity-Check Codes for Modulation and Detection", ten Brink, Kramer & Ashikhmin

"Design of Repeat-Accumulate Codes for Iterative Detection & Decoding", ten Brink & Kramer



Gerhard lecturing at TU Wien

All photos by Jossy Sayir

Stephan ten Brink invented EXIT charts as part of his PhD thesis at the University of Stuttgart

Stephan in Vienna

EXIT Charts for AWGN Channels

- J-function: $C_{AWGN} = J(\sigma_{ch})$
  (defined in Lecture 11)

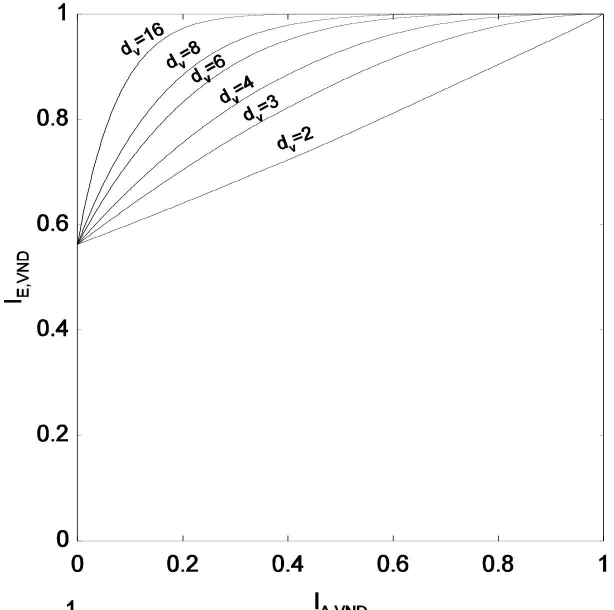
- VND: $I_E(I_A) = J\left(\sqrt{(d_v - 1)[J^{-1}(I_A)]^2 + \sigma_{ch}^2}\right)$

  (applying information combining as in lecture 11)

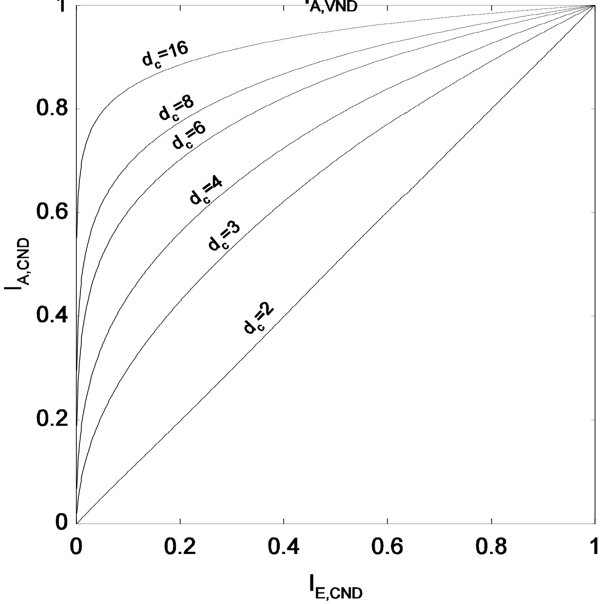
- CND: $I_E(I_A) \approx 1 - J\left(\frac{J^{-1}(1 - I_E)}{\sqrt{d_c - 1}}\right)$

  (based on duality approximation, see Ashikhmin, ten Brink & Kramer “Extrinsic Transfer Functions: Model and Erasure Channel Properties”)

AWGN Channels

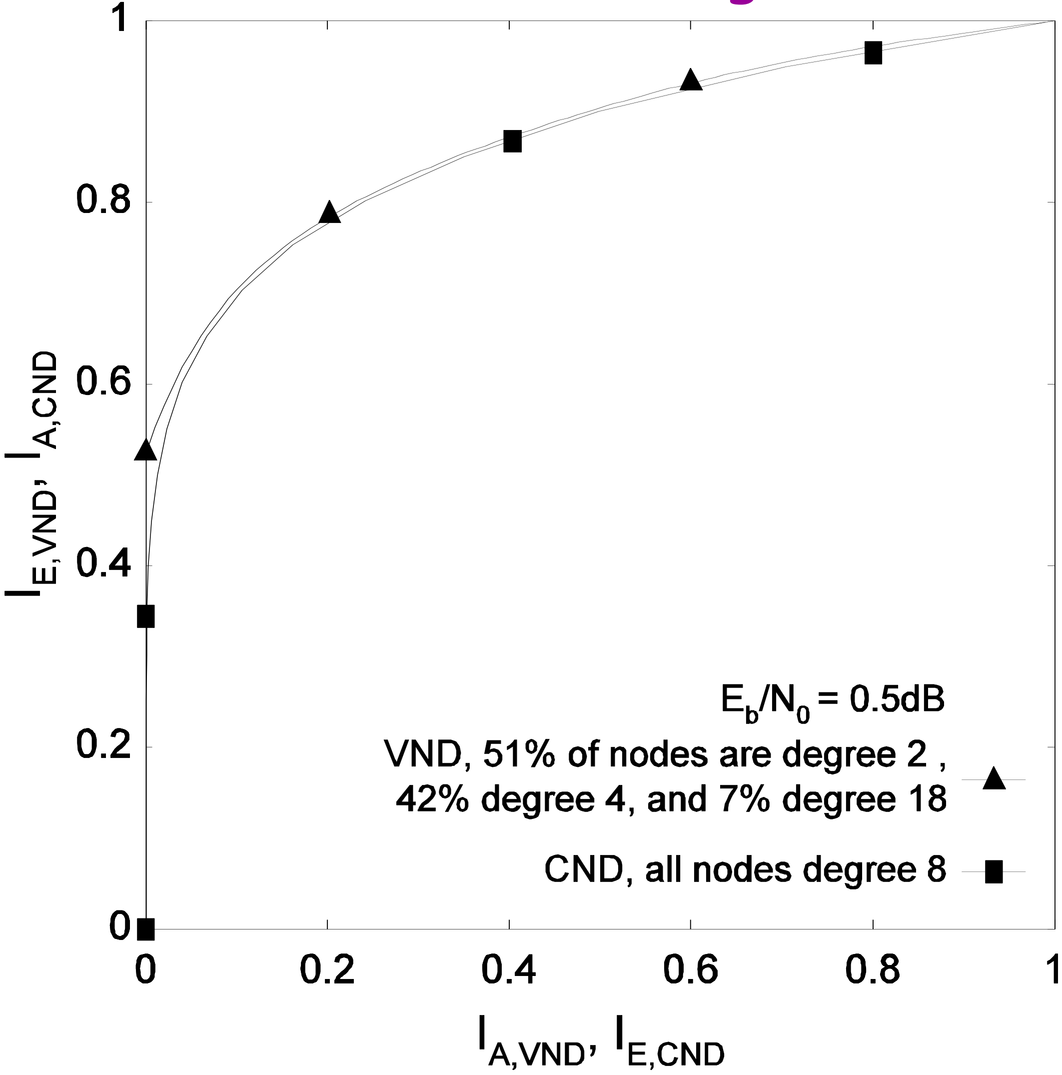


Variable Nodes



Check Nodes

Curve fitting



Figures from paper by ten Brink & al.

MIMO Fading Channels

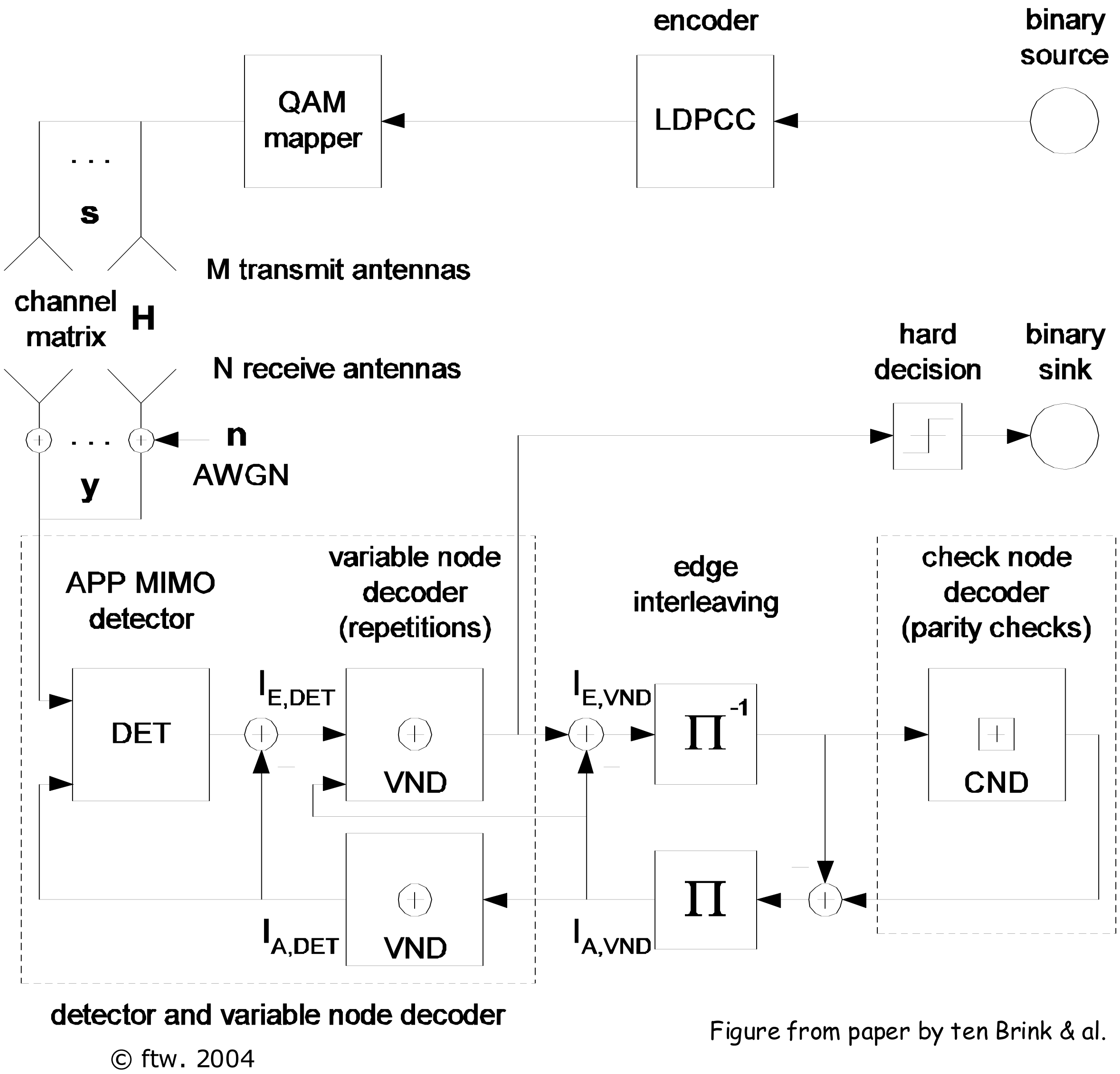


Figure from paper by ten Brink & al.

The detector becomes part of the variable node decoder!

MIMO Fading channels

- CND transfer curve remain unchanged

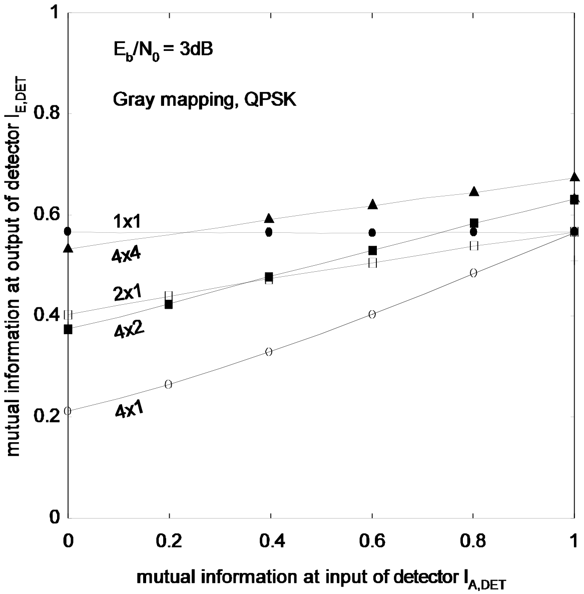
- VND transfer curves are computed by cascading

$$I_{A1} \rightarrow I_{A2} = J\left(\sqrt{d_v} J^{-1}(I_{A1})\right) \rightarrow I_{A3} = I_{E,DET}(I_{A2})$$

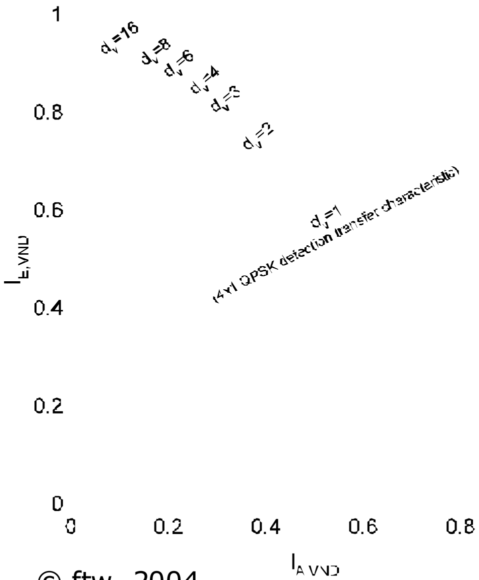
$$\rightarrow I_E = J\left(\sqrt{(d_v - 1)[J^{-1}(I_{A1})]^2 + [J^{-1}(I_{A3})]^2}\right)$$

- $I_{E,DET}(I_{A,DET})$ can only be measured by simulation. For the curve fitting (tuning the variable node degrees), it is convenient to approximate the transfer curve of the detector by a polynomial

MIMO Fading Channels

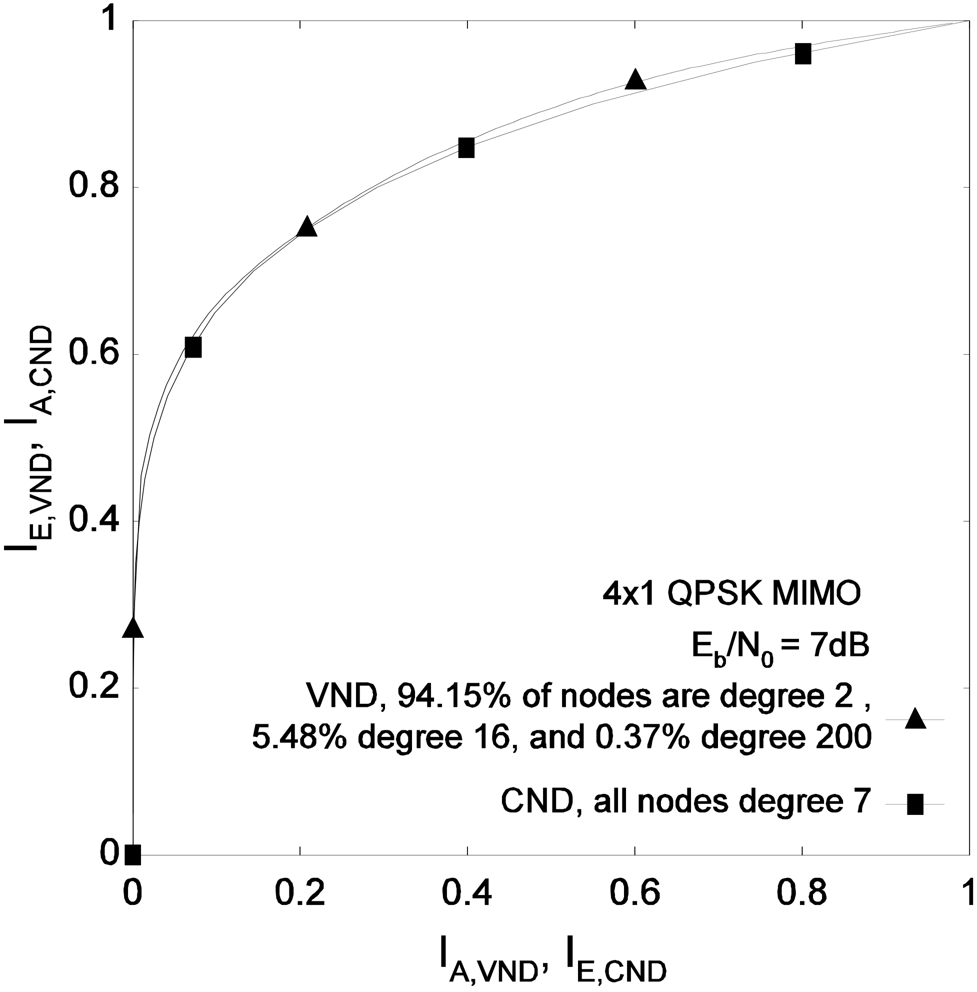


Detector Curves



Combined Curves

Curve fitting for MIMO



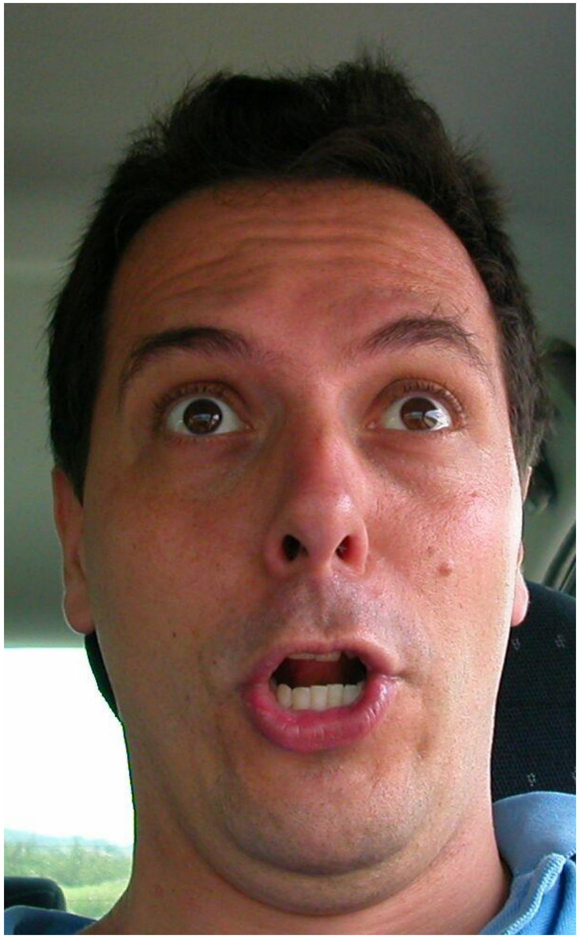
Figures from paper by ten Brink & al.

MIMO Fading: Performance

- 1.25 dB from the Ergodic Capacity of MIMO fading channels
- Up to 6 dB better than iterative turbo coding & detection
- 6-8 dB better than space-time coding based approaches
- Similar design achieve comparable results based on Repeat-Accumulate Codes

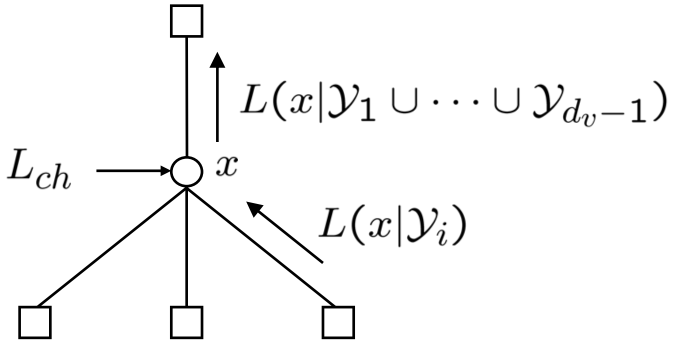
Sub-optimal Decoders

"Improved Sum-Min Decoding of LDPC Codes",
Lechner & Sayir, to be presented at ISITA 2004



Sum-Product Algorithm

Variable Nodes



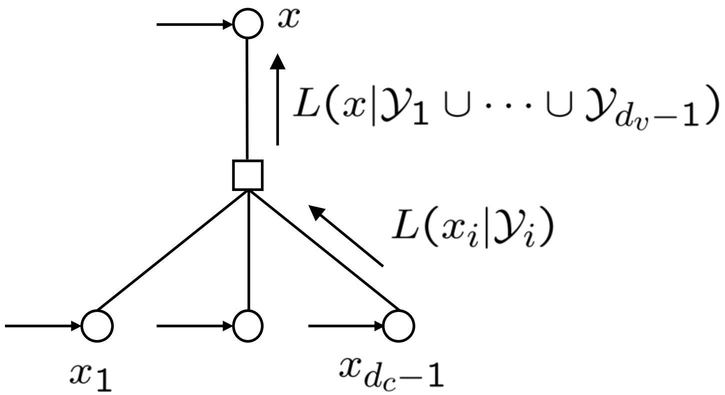
Repeat Code

$$L(x|\mathcal{Y}_1 \cup \cdots \cup \mathcal{Y}_{d_v-1}) =$$

$$= L_{ch} + \sum_i L(x|\mathcal{Y}_i)$$

Just a sum... Easy.

Check Nodes



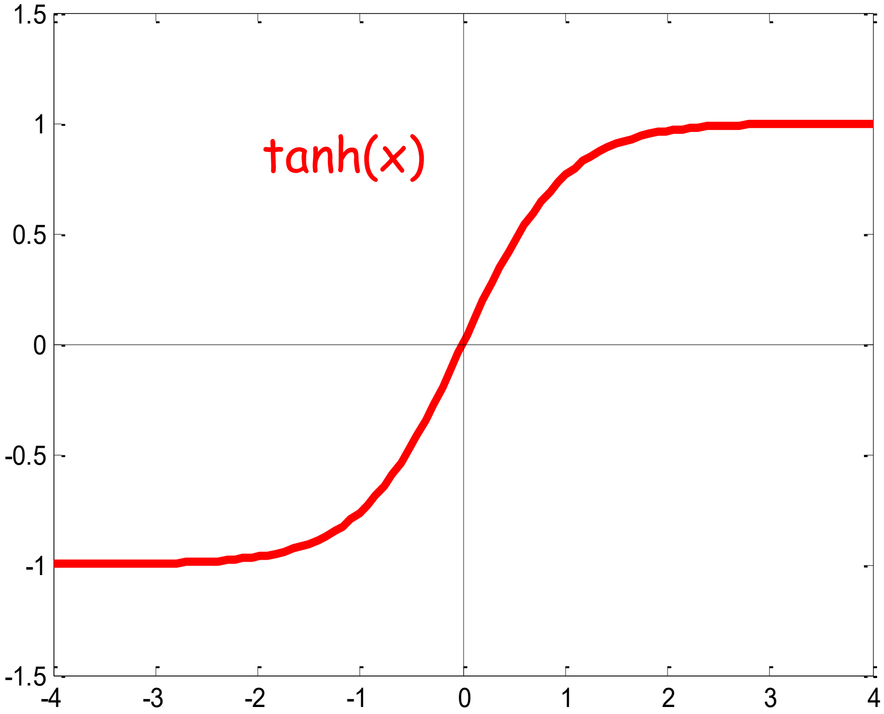
Single Parity-Check Code

$$L(x|\mathcal{Y}_1 \cup \cdots \cup \mathcal{Y}_{d_v-1}) =$$

$$= 2 \cdot \tanh^{-1}\left[\prod_i \tanh \frac{L(x_i|\mathcal{Y}_i)}{2}\right]$$

Complicated!!!!

$\tanh^{-1} \prod \tanh x_i$



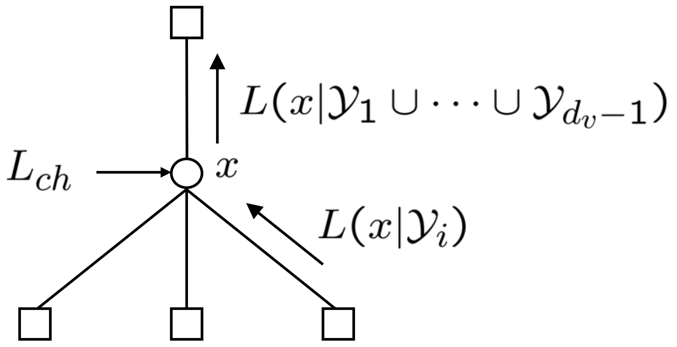
➔ the large x_i will have little impact on the product

➔ the product is well approximated by:

$$\tanh^{-1} \tanh(\min x_i) = \min x_i$$

Sum-Min Algorithm

Variable Nodes



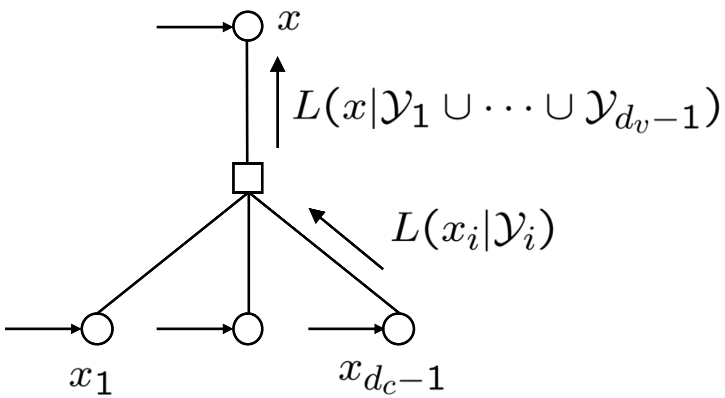
Repeat Code

$$L(x|\mathcal{Y}_1 \cup \cdots \cup \mathcal{Y}_{d_v-1}) =$$

$$= L_{ch} + \sum_i L(x|\mathcal{Y}_i)$$

Same as before...

Check Nodes



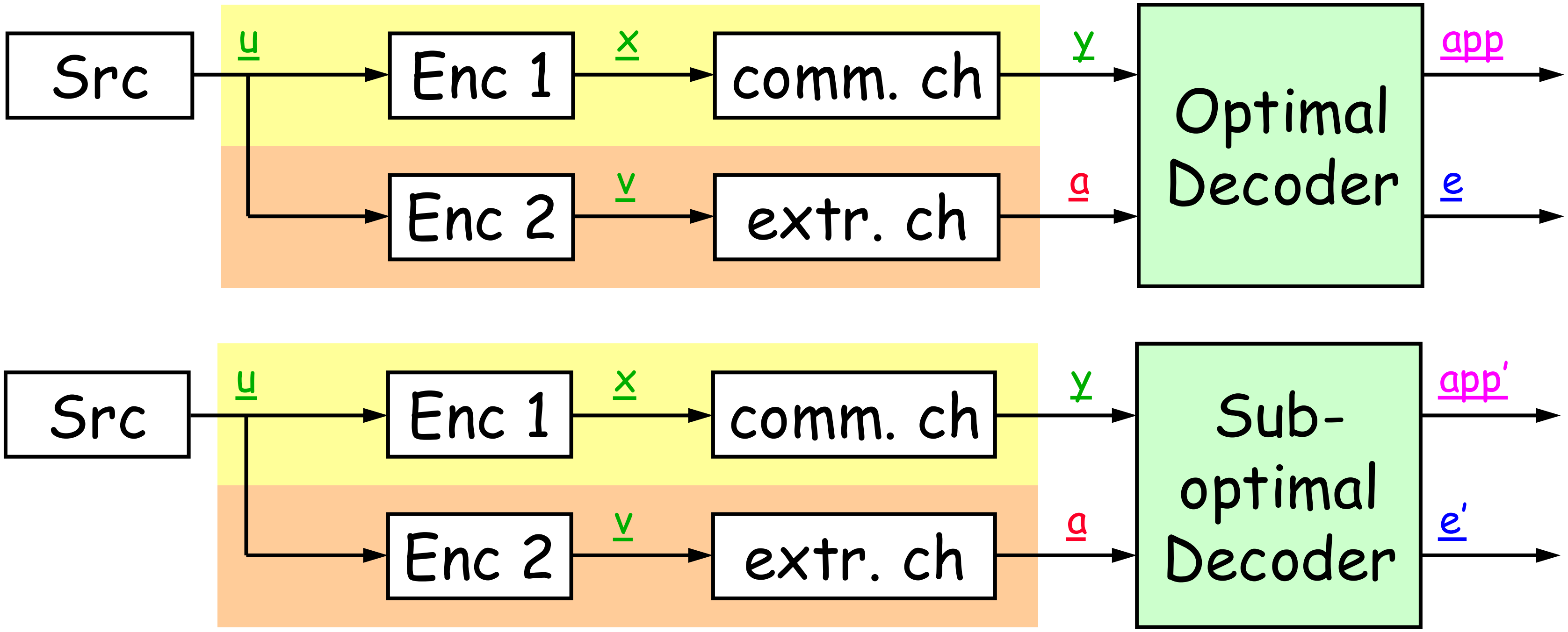
Single Parity-Check Code

$$L(x|\mathcal{Y}_1 \cup \cdots \cup \mathcal{Y}_{d_v-1}) =$$

$$\approx \min_i L(x_i|\mathcal{Y}_i)$$

Simplified.

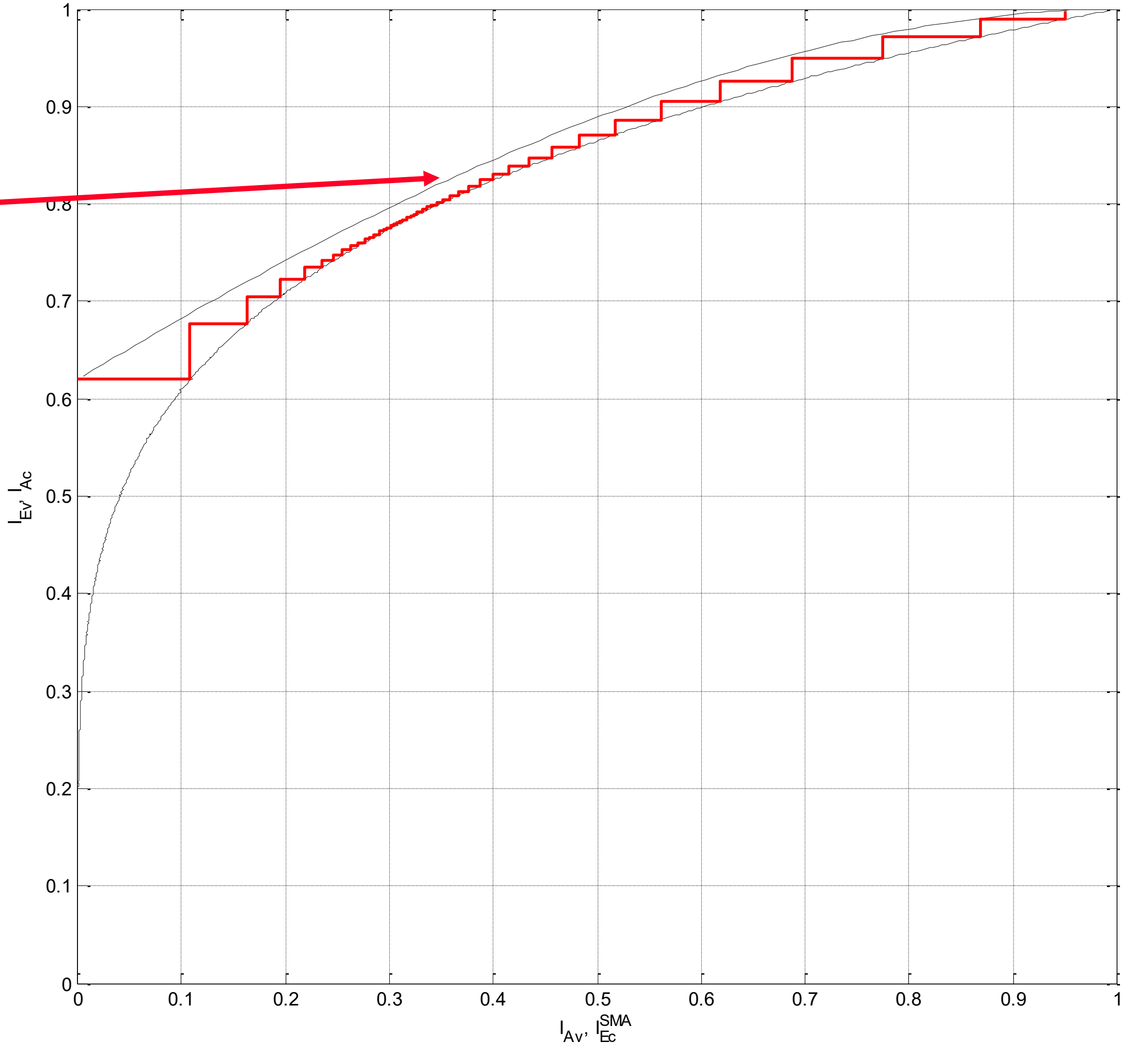
Good News…



$$I(v\,;\,app) = I(v\,;\,a\ y) > I(v\,;\,app')$$

But : the check-node EXIT function (measured by simulation) of the min is only negligibly lower than that of the arctanh^{-1}!

Bad News…



the trajectory is not on the check node curve…

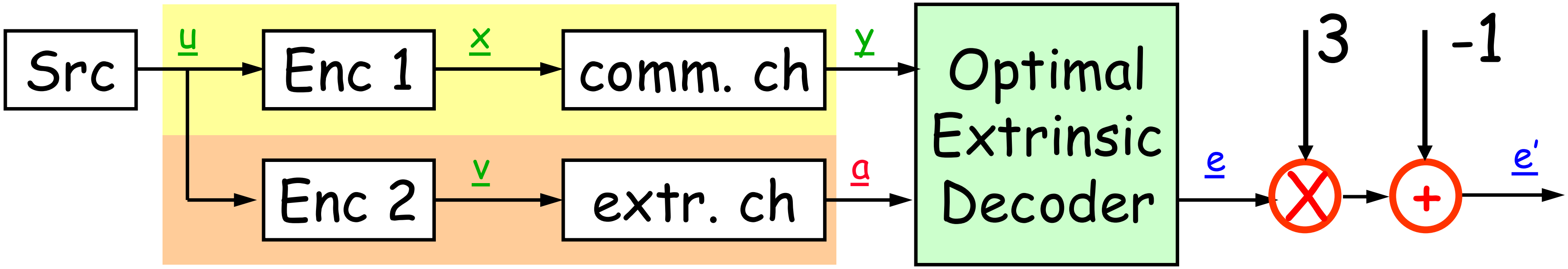
Sum-Min Algorithm

- The performance of the sum-min algorithm is far worse than that predicted by EXIT chart analysis
- Density Evolution predicts the bad performance of the sum-min algorithm accurately

➔ are EXIT charts unsuitable for sub-optimal decoders?

➔ is something wrong with the sum-min algorithm?

Something to think about…



➔ the resulting decoder is not optimal anymore

➔ On the other hand, the mutual information remains unaffected by these transformations, i.e., $I(\underline{v} ; \underline{e}) = I(\underline{v} ; \underline{e}')$

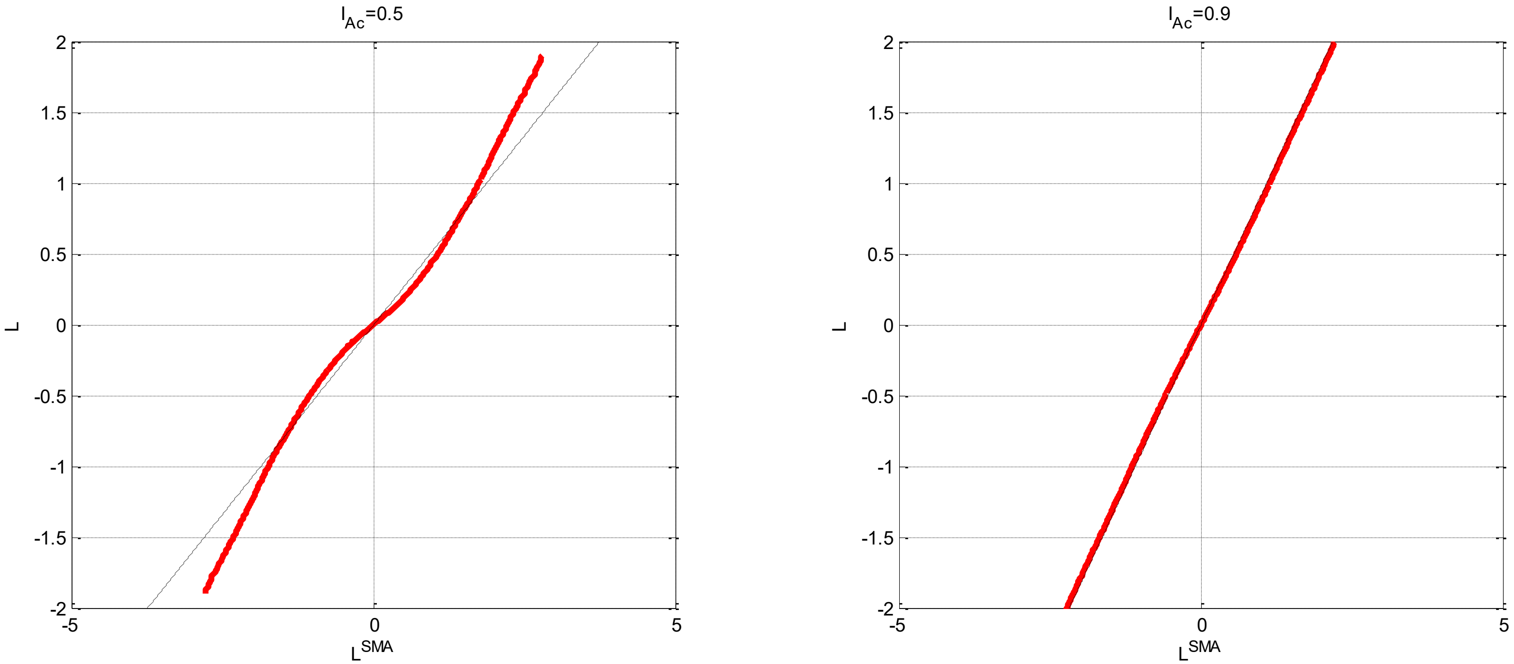
The problem with the sum-min algorithm

- Something similar is happening in our sum-min algorithm
- The min has almost as much mutual information to the code bits as has the arctanh^{-1}
- However, the arctanh^{-1} is an L-value, while the min is not!
- We need to apply a post-processing real-valued function to the min before forwarding it to the variable node!

Post-processing

We compute an L-value based on the min:

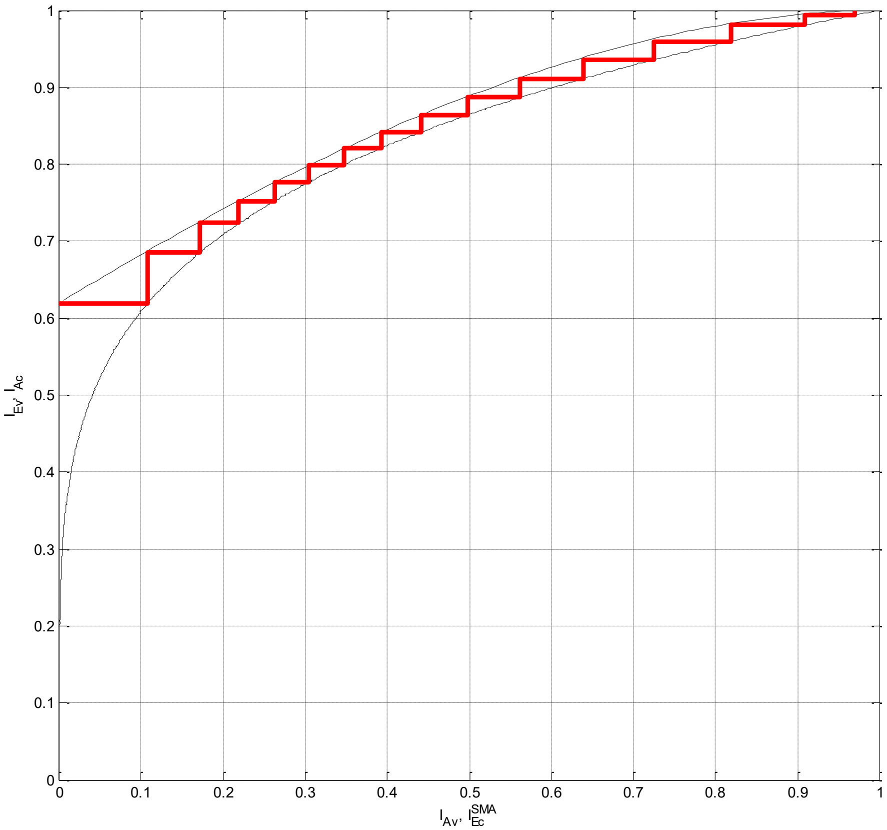
$$L\left(x|minL_a(x)\right) = \log\frac{[p(z)+p(-z)]\cdot\left[\phi_+(z)+\phi_-(z)\right]^{d_c-2}+[p(z)-p(-z)]\cdot\left[\phi_+(z)-\phi_-(z)\right]^{d_c-2}}{[p(z)+p(-z)]\cdot\left[\phi_+(z)+\phi_-(z)\right]^{d_c-2}-[p(z)-p(-z)]\cdot\left[\phi_+(z)-\phi_-(z)\right]^{d_c-2}}$$



Postprocessing functions for I_a=0.5 and for I_a = 0.9

Postprocessing almost linear…

With post-processing



Performance of Sum-Min almost as good to performance of Sum-Product (0.1 dB loss)

Information Combining

From Ingmar Land's homepage

Ingmar combining information

Ingmar Land came up with information combining as a way to bound EXIT curves for general binary-input symmetric channels. He is about to finish his PhD at Uni Kiel with Prof. P. Hoeher.

Information Combining Bounds

Exam

Pick a card at random. The card will have one topic from each row in the following table:

1.	Linear Codes	Information Theory	LDPC Codes	Easy
2.	Turbo Codes	EXIT Charts	LDPC Code Design	Medium
3.	Joker			Medium to difficult

You may leave out 1 of the 3 topics. The exam will last 30 minutes, i.e., 15 minutes per topic.

Joker: read one paper in the following page and discuss its contents for 15 minutes.

List of papers

Please pick 1 (and only 1) of the following papers and read it if you wish to prepare for the joker exam question.

Richardson & Urbanke, "The Capacity of Low-Density Parity-Check Codes Under Message-Passing Decoding	Information-Theory Transactions, September 2001	Density Evolution ex´plained. MEDIUM.
Luby, Mitzenmacher, Shokrollahi, "Analysis of Random Processes via And-Or Tree Evaluation"	http://www.eecs.harvard.edu/~michaelm/NEWWORK/postscripts/andor-conf.pdf	Convergence analysis for the BEC, MEDIUM.
Kshischang, Frey & Loeliger, "Factor Graphs and the Sum-Product Algorithm",	Information-Theory Transactions, September 2001	Everything you always wanted to know about factor graphs. LONG.
Ashikhmin, Kramer & ten Brink, "Extrinsic Information Transfer Functions: Model and Erasure Channel Properties"	http://cm.bell-labs.com/cm/ms/who/gkr/	EXIT charts explained for the BEC. EASY.
ten Brink & Kramer, "Design of Low-Density Parity-Check Codes for Modulation and Detection"		EXIT chart design for MIMO fading channels. MEDIUM.
ten Brink & Kramer, "Design of Repeat-Accumulate Codes for Iterative Detection and Decoding"		EXIT chart design of RA codes. MEDIUM.
Land, Huettinger, Hoeher & Huber, "Bounds on information combining"	http://www-lns.tf.uni-kiel.de/ict/il/publications.html	The theory behind bounds on the EXIT curves. MEDIUM.
Lechner & Sayir, "Improved Sum-Min Decoding of LDPC Codes"	email sayir@ftw.at or lechner@ftw.at	Sub-optimal decoding and EXIT charts. MEDIUM.

Thank you for attending our lecture

Jossy & Gottfried