Theory and Design of

Turbo and Related Codes

Lecture 11

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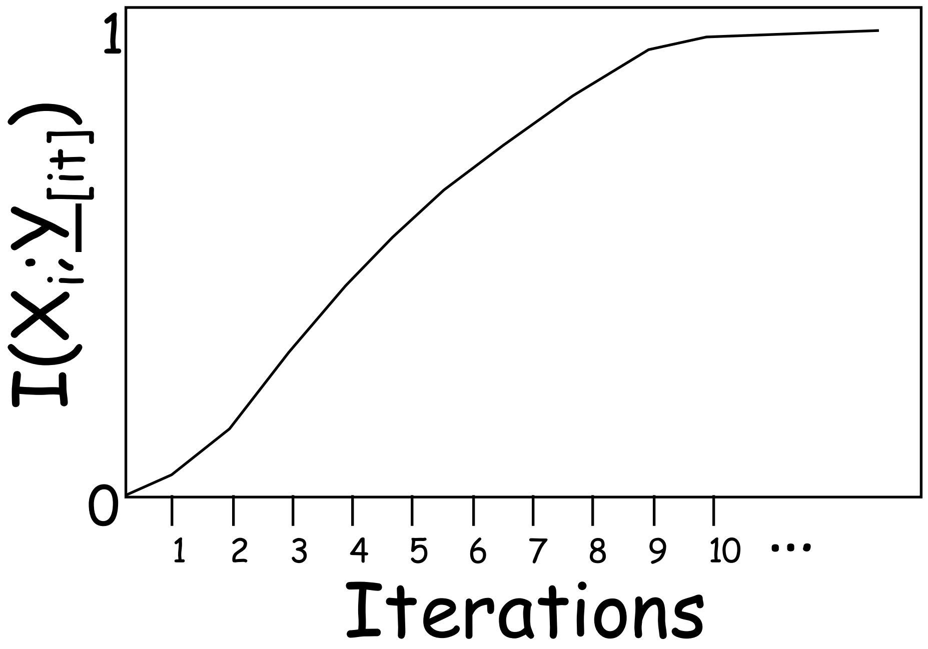
http://userver.ftw.at/~jossy/turbo/index.html

Kplus
Kompetenzzentren-Programm

Last Lecture



increasing iterations $\Rightarrow$ increasing observations

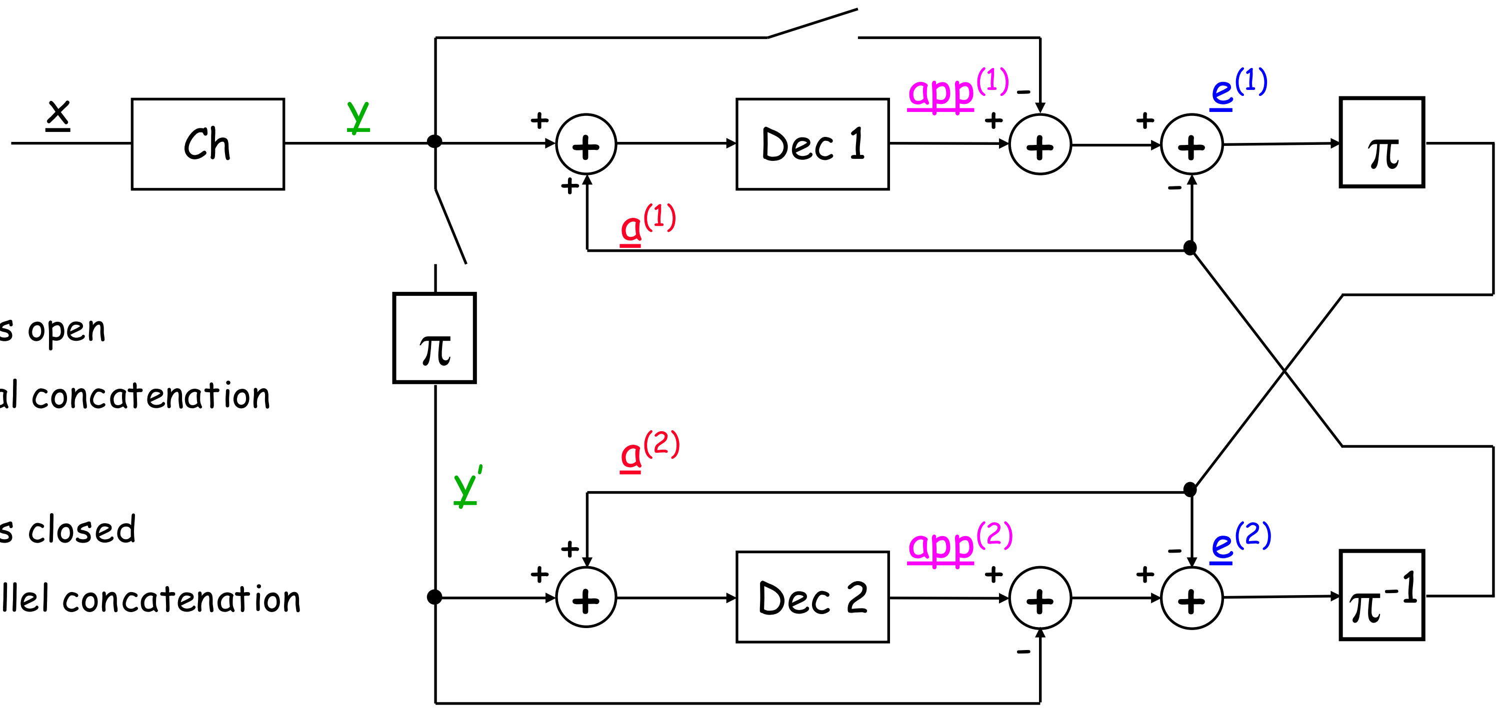
$$I(X_i;\underline{Y}_{[it]}) \rightarrow 1 \Rightarrow P_b \rightarrow 0$$

$I(X_i;\underline{Y}_{[it]}) \rightarrow 1$???

Evolution of $I(X_i;\underline{Y}_{[it]})$

It is a hard problem to analyze the evolution of this quantity for a decoder.

Iterative decoders exchange messages (extrinsic and a-priori). What about “tracking“ those messages…?



switches open

→ serial concatenation

switches closed

→ parallel concatenation

Tracking of Messages

Tracking of messages would mean tracking of pdfs.

Instead of tracking the pdfs we reduce the problem to tracking of mutual information between the messages and the codeword which are scalar quantities.

$$I_A = \frac{1}{N} \sum_{n=1}^{N} I(X_i; A_i)$$

$$I_E = \frac{1}{N} \sum_{n=1}^{N} I(X_i; E_i)$$

$$I_A^{(1)} = 0$$

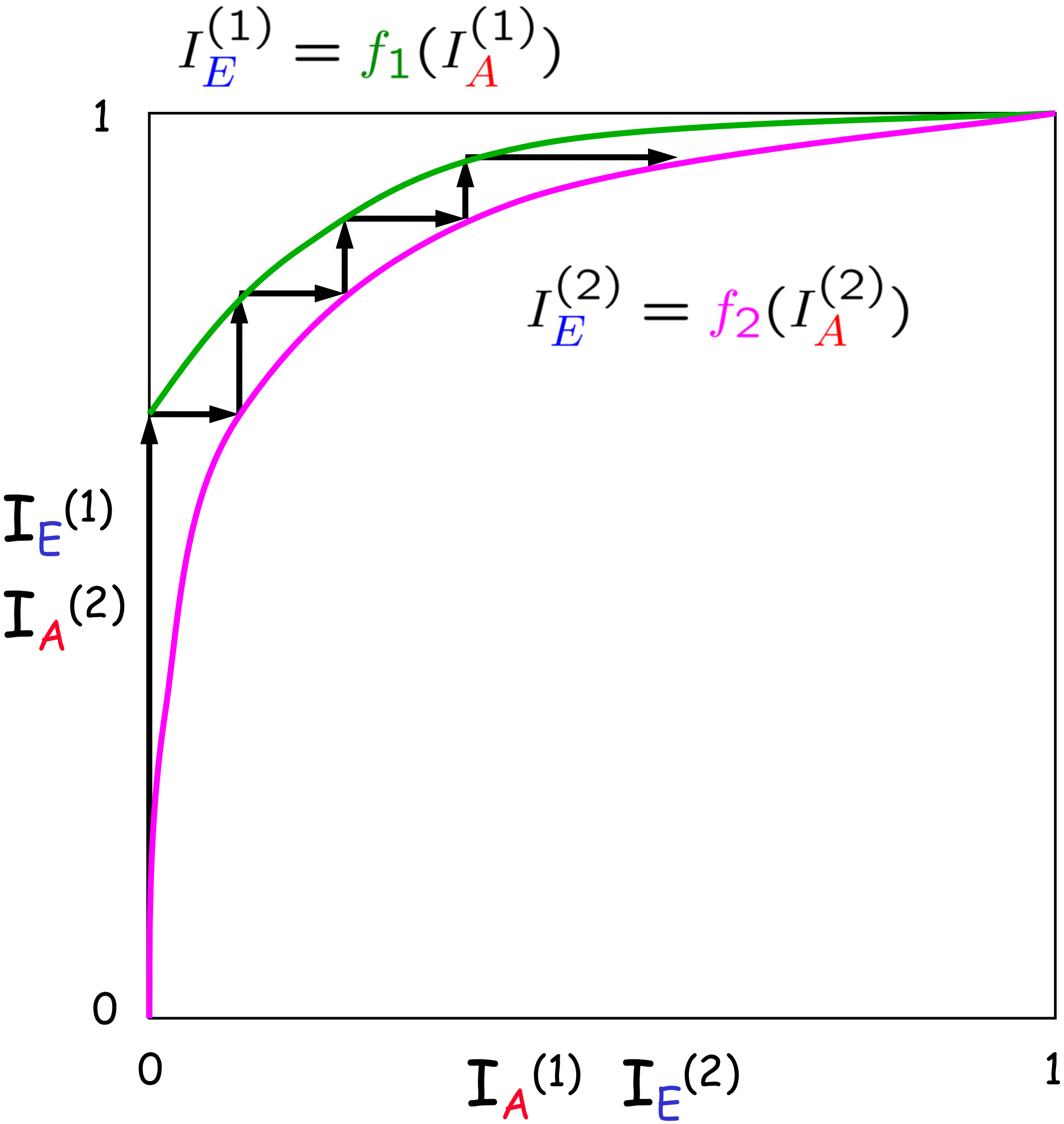
$$\Rightarrow I_E^{(1)} \rightarrow I_A^{(2)}$$

$$\Rightarrow I_E^{(2)} \rightarrow I_A^{(1)}$$

$$\cdots$$

I_A, I_E ..... average symbolwise mutual information

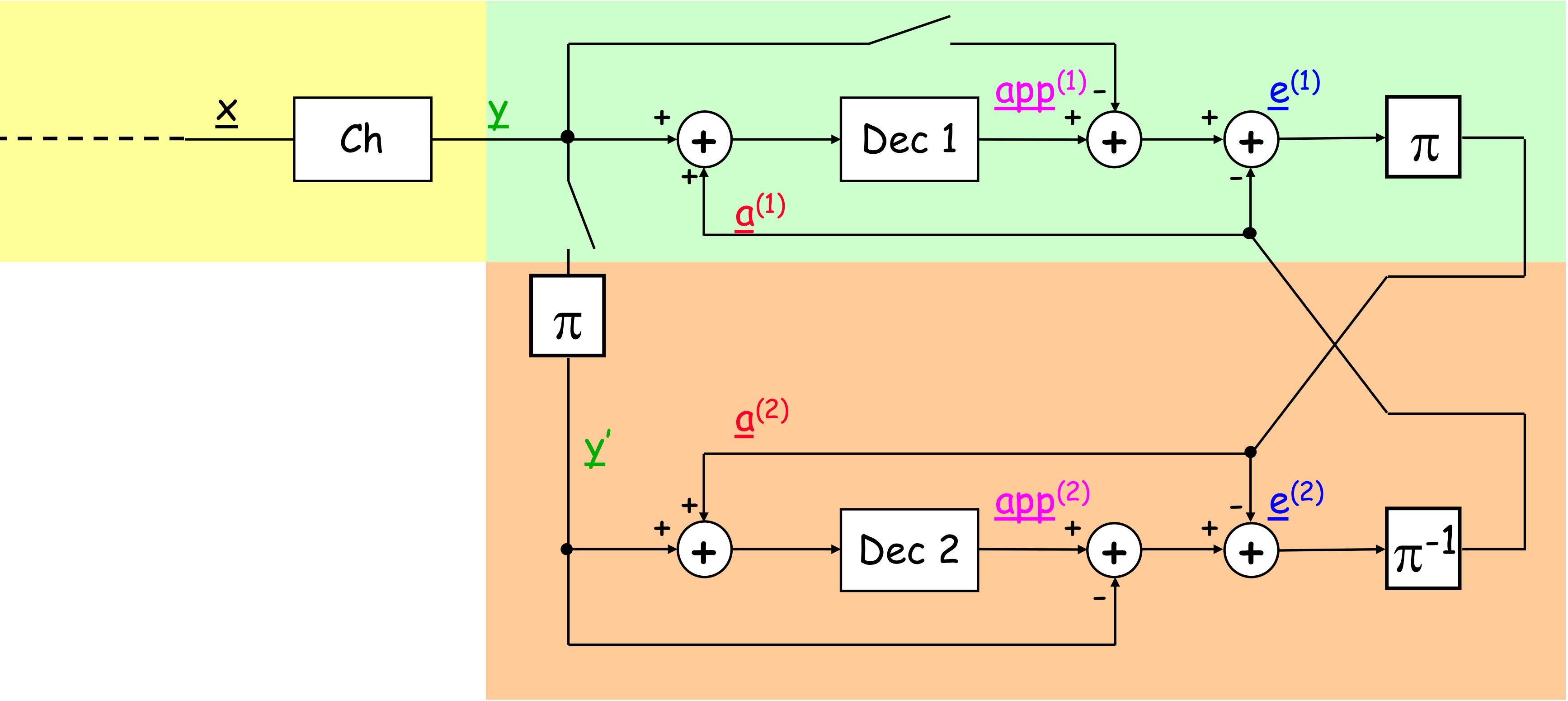
Tracking of Messages

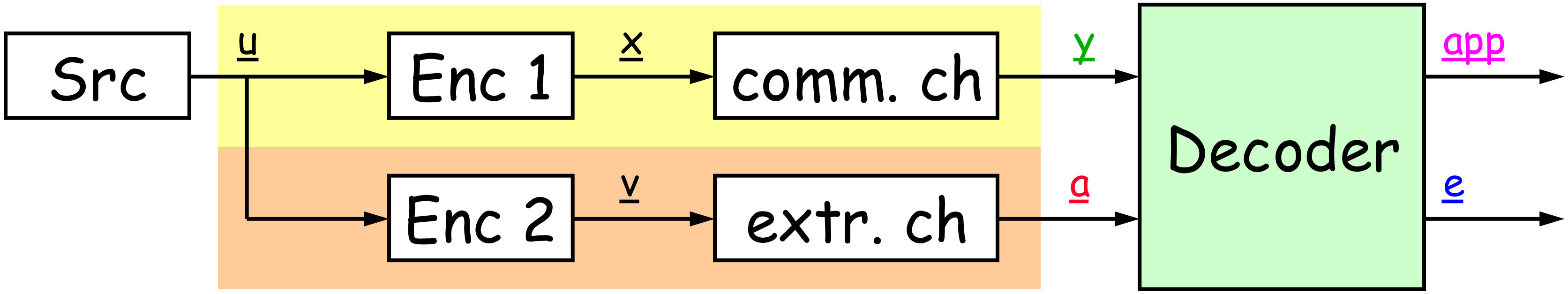


This assumes that the decoder depends only on mutual information!

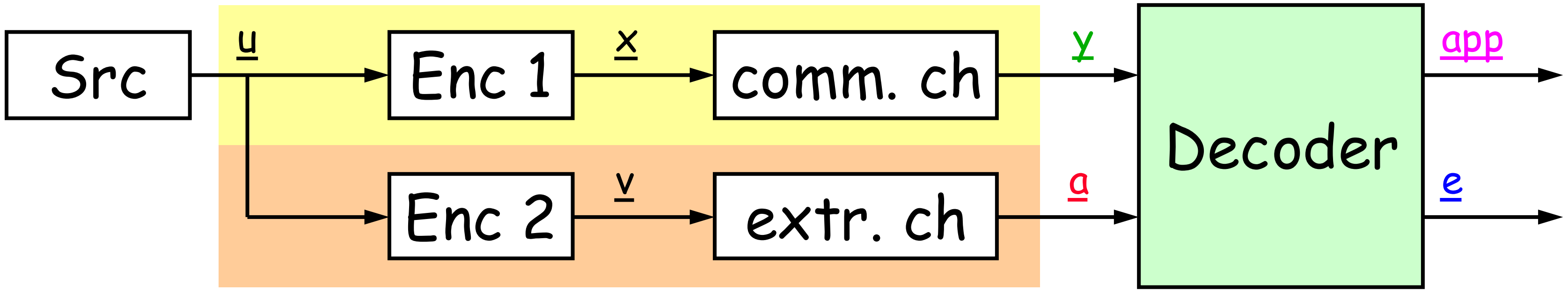
Problem:
How to compute the "transfer functions" f_1 and f_2?

Extrinsic Channel Model





Extrinsic Channel Model



A-priori messages are modeled as independent noisy observations of the encoded source.

Assumptions:

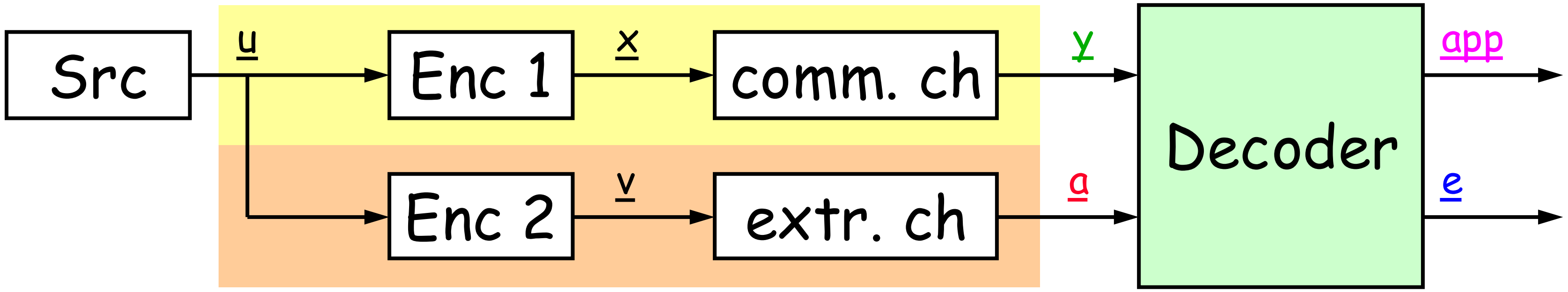
- independent observations
- model for extrinsic channel

$$I_A = I(X_i; A_i) = I(V_i; A_i)$$

$$I_E = I(X_i; E_i) \leq I(X_i; \underline{Y}\,\underline{A}_{\backslash i})$$

with equality if the decoder is optimal

Transfer Functions

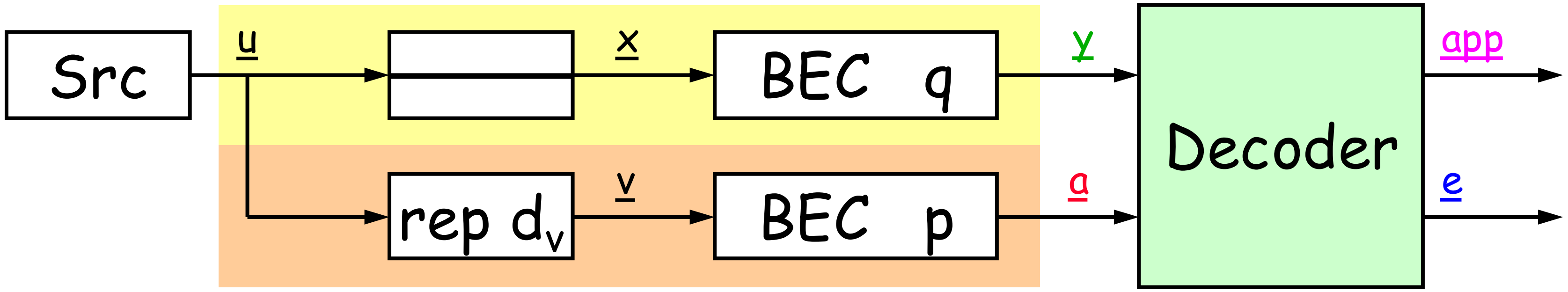


Assuming a model for the extrinsic channel we can vary I_A by varying the channel parameter.

At the output of the decoder we can measure/calculate I_E $\Rightarrow I_E = f(I_A)$

This is only exact if the model of the extrinsic channel is correct!

Variable Nodes and BEC

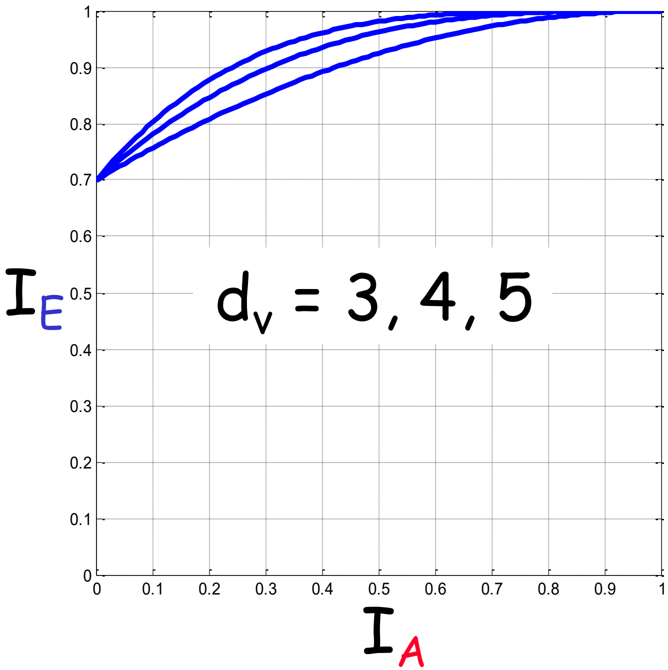


Extrinsic channel is modeled as BEC (exact).

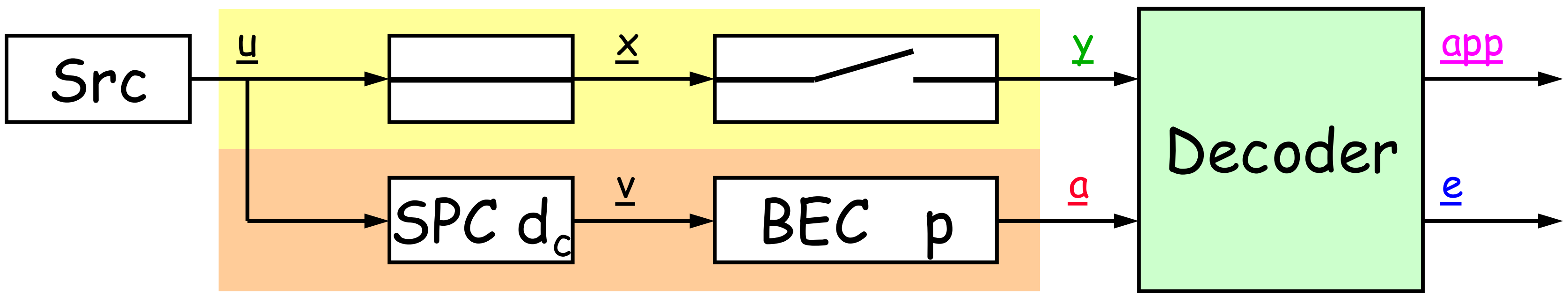
$$I_A = I(X_i; A_i) = I(V_i; A_i) = 1 - p$$

$$I_E = I(X_i; \underline{Y A_{\setminus i}}) = 1 - qp^{d_v - 1}$$

$$I_E = 1 - q(1 - I_A)^{d_v - 1}$$



Check Nodes and BEC

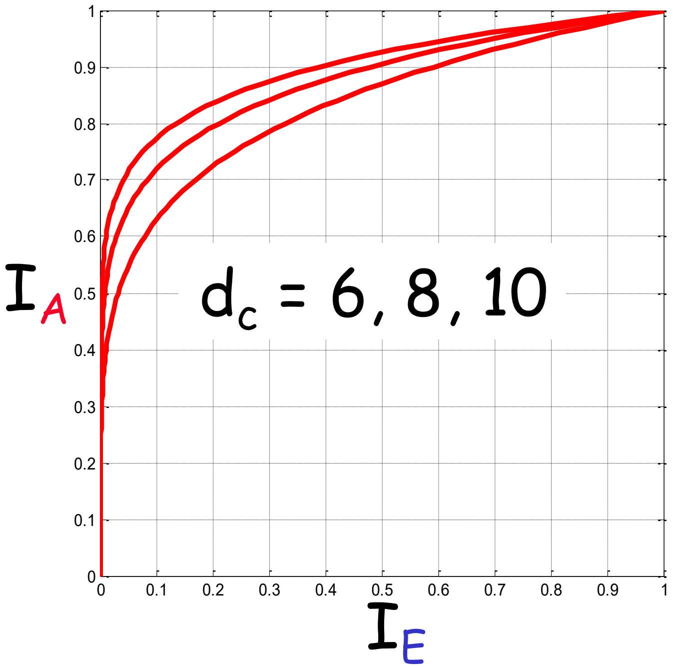


SPC ... single parity check

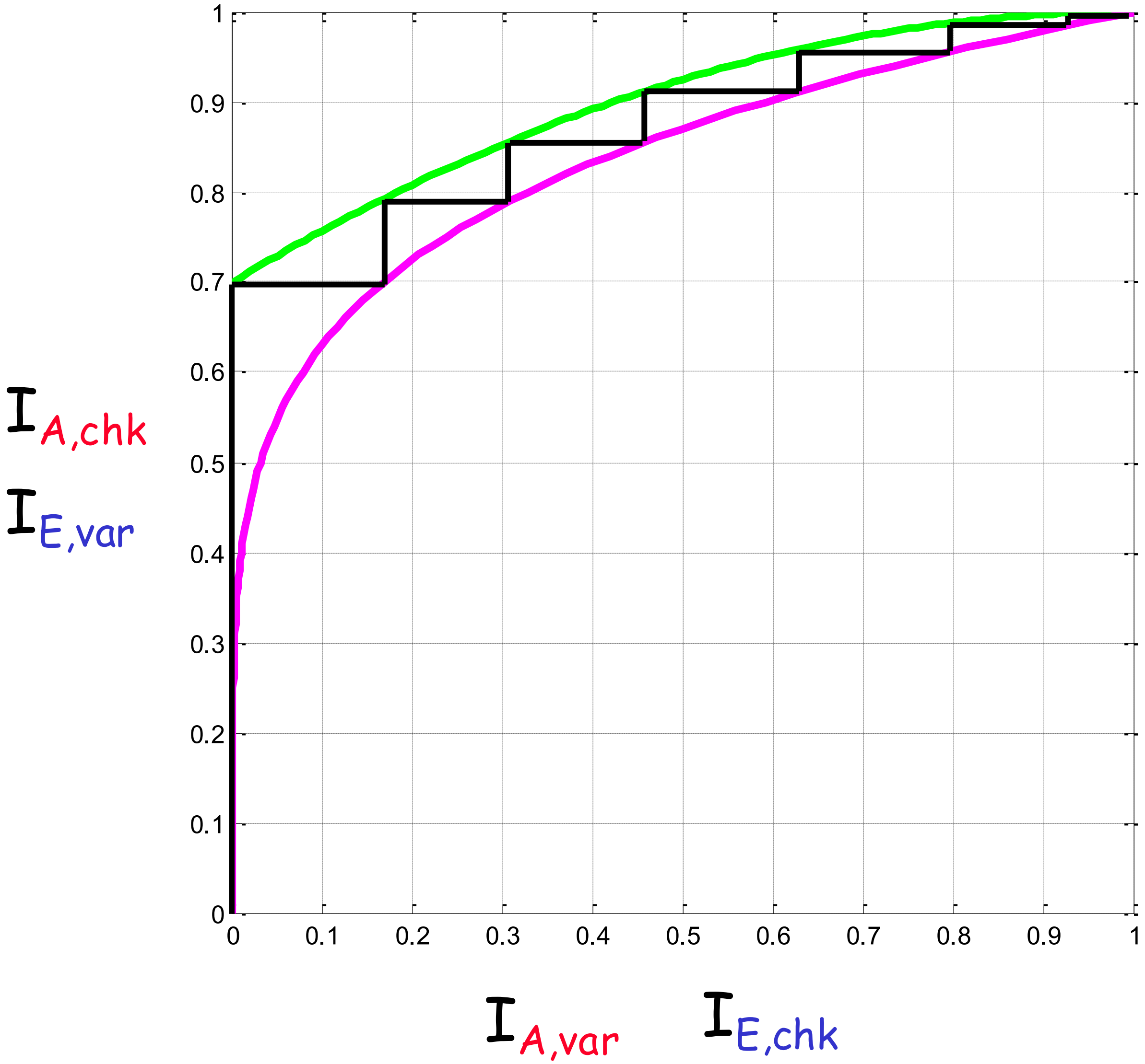
$$I_A = I(X_i; A_i) = I(V_i; A_i) = 1 - p$$

$$I_E = I(X_i; \underline{Y A_{\setminus i}}) = (1-p)^{d_c - 1}$$

$$I_E = (I_A)^{d_c - 1}$$



EXIT Chart of LDPC Code



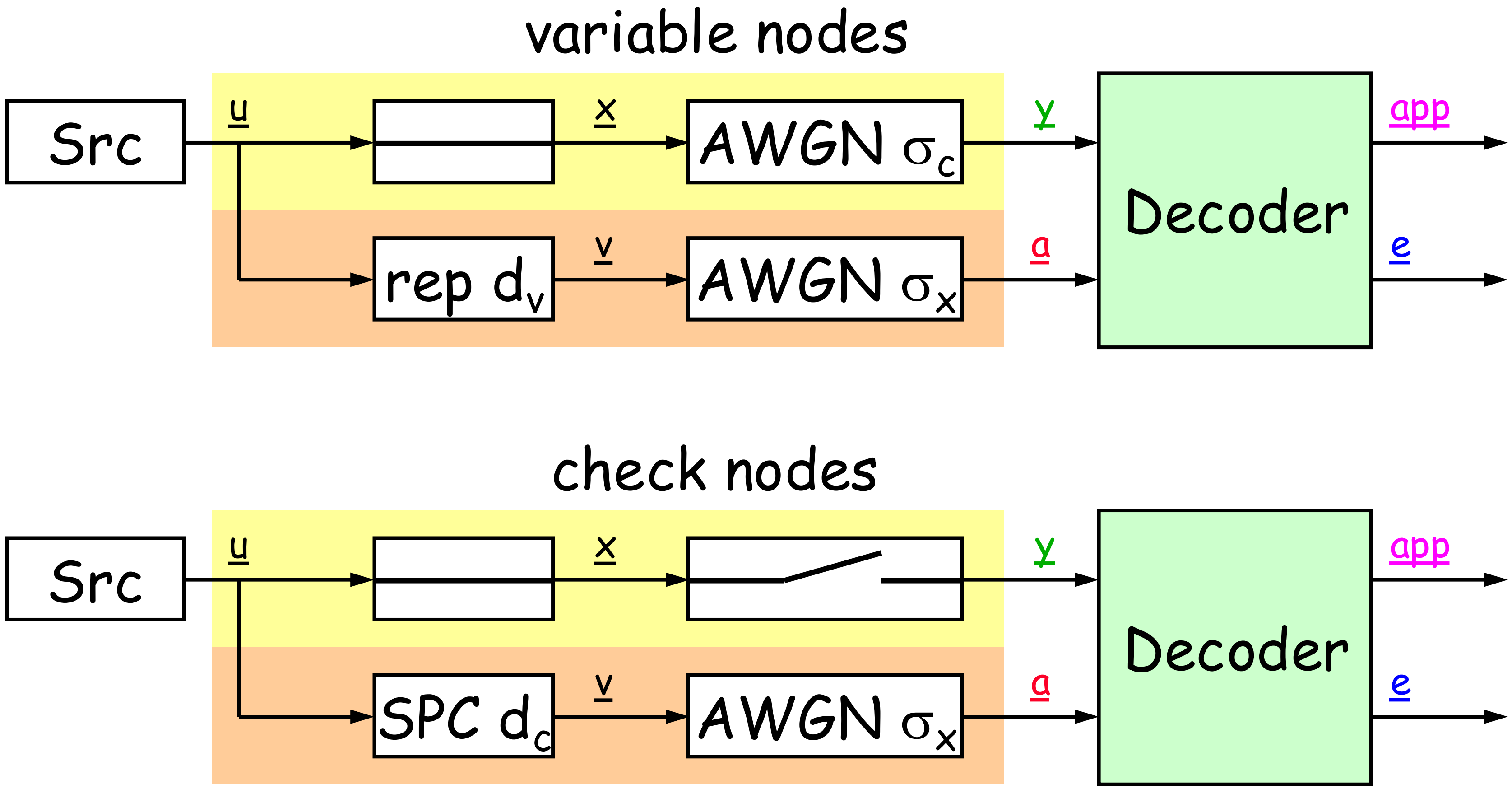
Other Channels

Modeling the extrinsic channel as a BEC is exact if the communication channel is a BEC.

For other communication channels, the assumption of the extrinsic channel is in general an approximation.

If the communication channel is an AWGN channel, we model the extrinsic channel also as an AWGN, but this is only an approximation!

AWGN Channel



Convolutional Codes

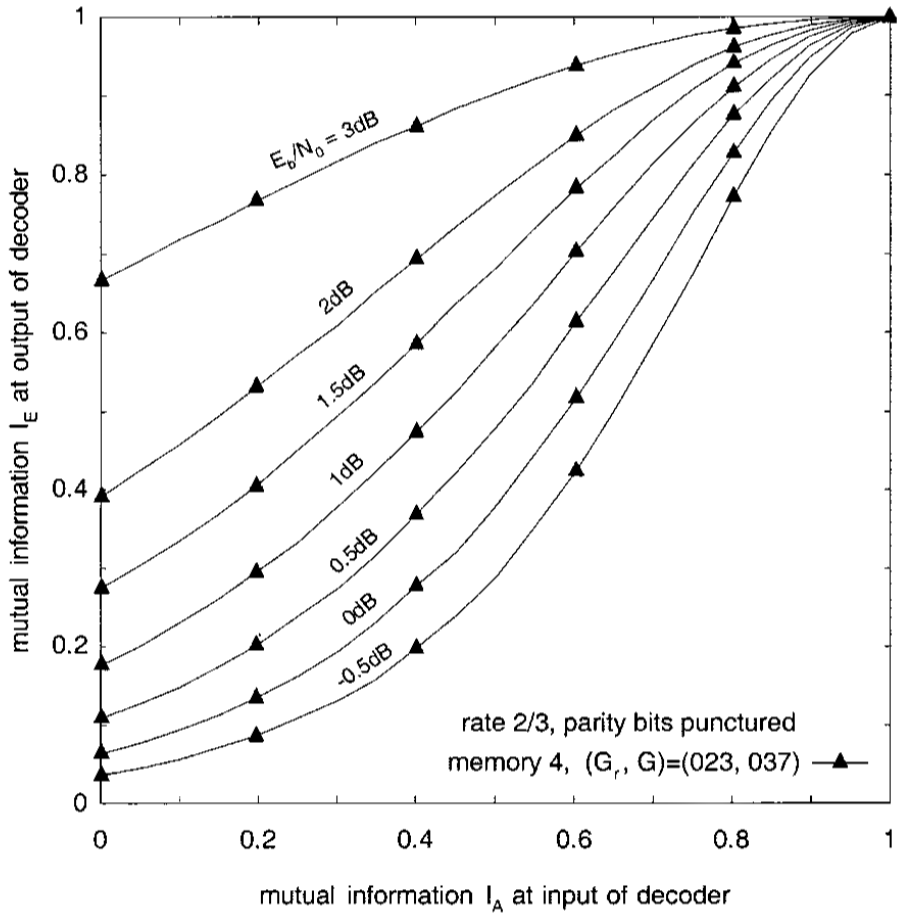
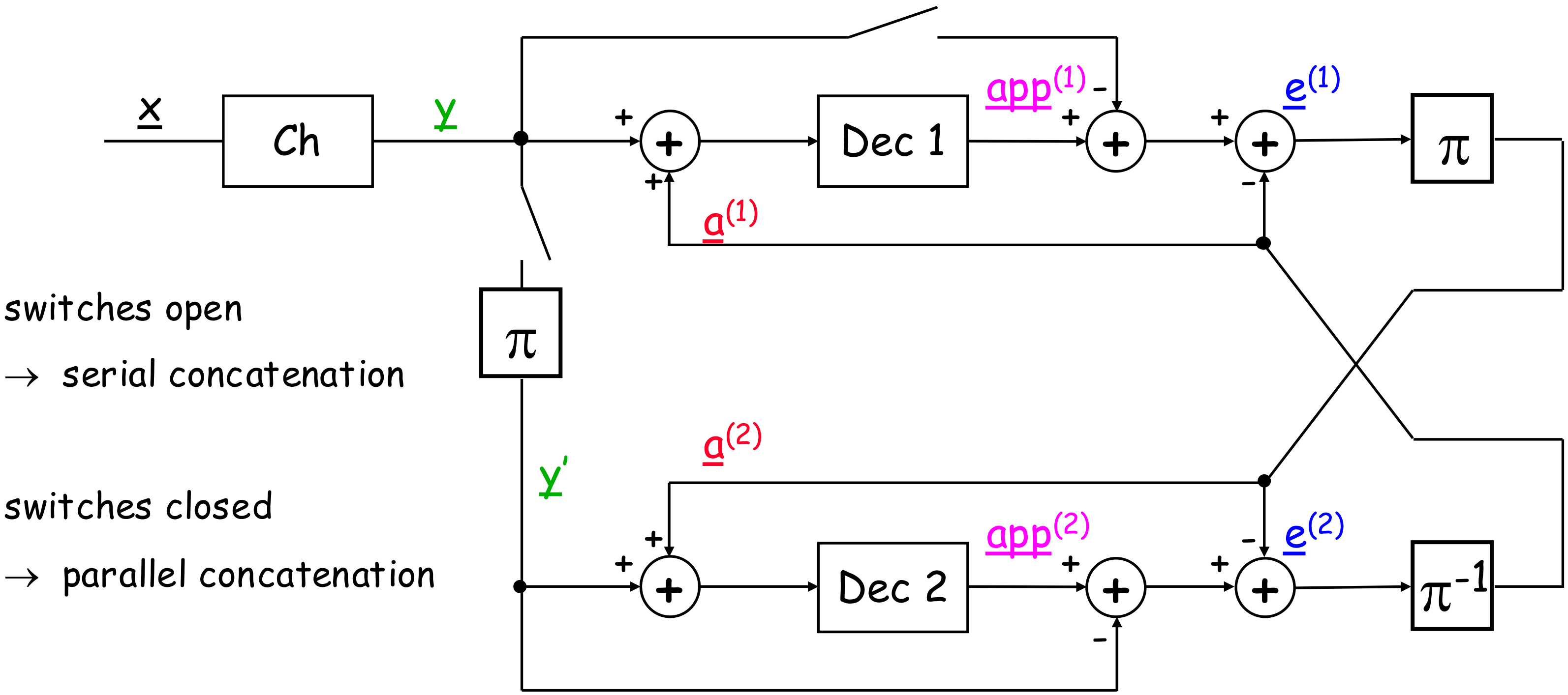


Fig. 2. Extrinsic information transfer characteristics of soft in/soft out decoder for rate 2/3 convolutional code; E_b/N_0 of channel observations serves as parameter to curves.

Stephan ten Brink, “Convergence Behavior of Iteratively Decoded Parallel Concatenated Codes”, IEEE Trans. Comm. October 2001

Serial / Parallel Concatenation



Serial concatenation:

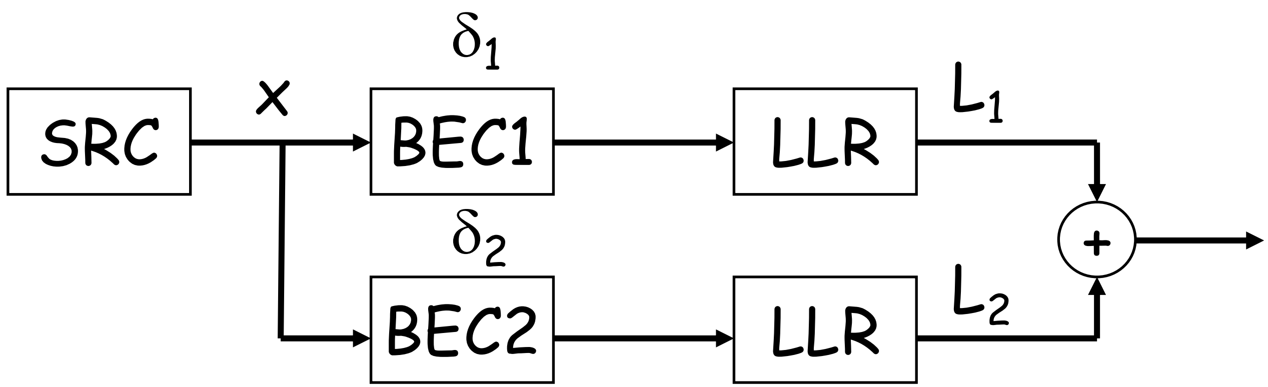
e = app - a

Parallel concatenation:

e = app - a – y

Information Combining BEC

What is the effect on mutual information when we add L-values?



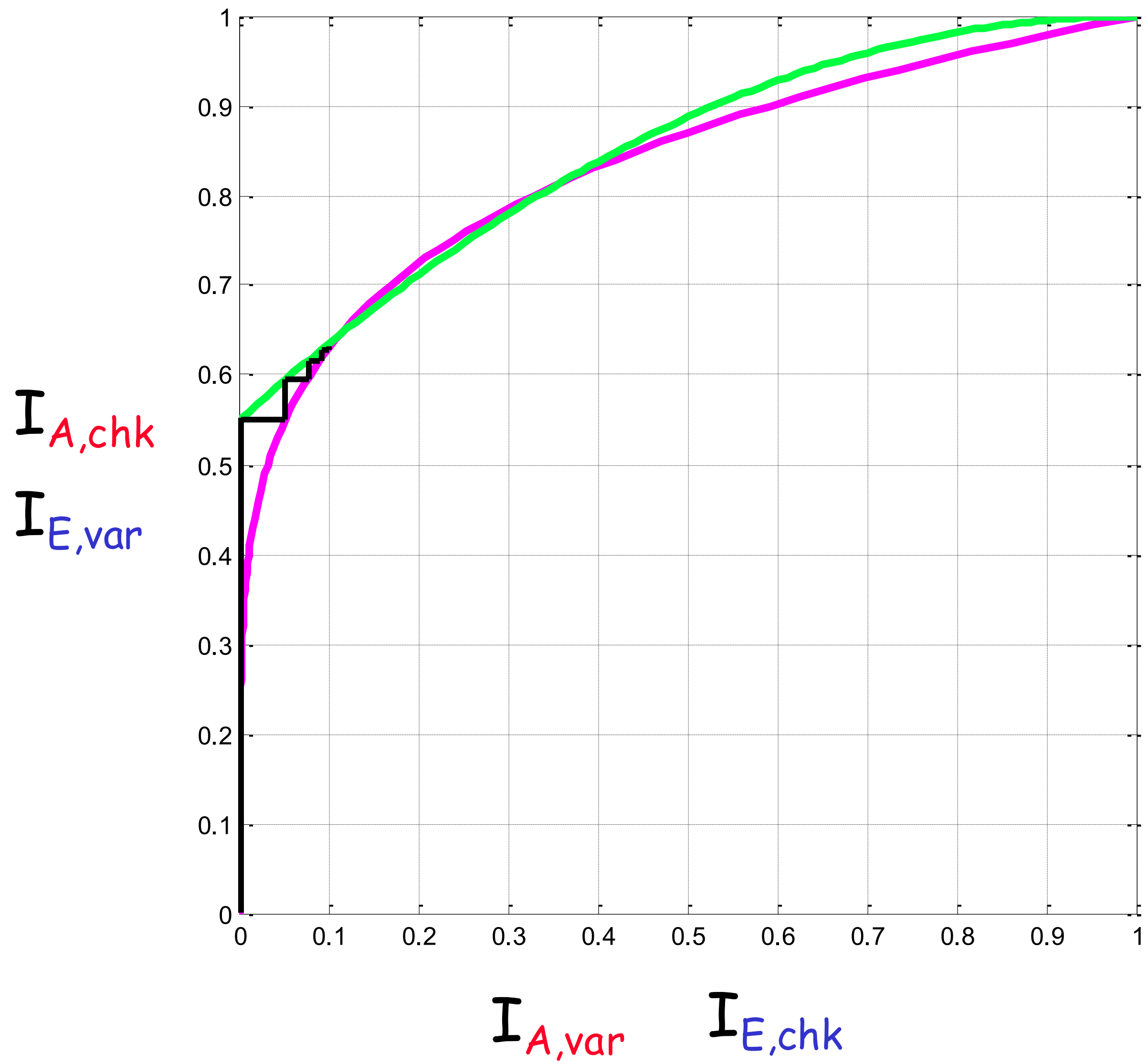
$I_1 = I(X;L_1) = 1 - \delta_1$

$I_2 = I(X;L_2) = 1 - \delta_2$

$I(X;L_1L_2) = 1 - \delta_1\delta_2$

$= 1 - (1-I_1)(1-I_2)$

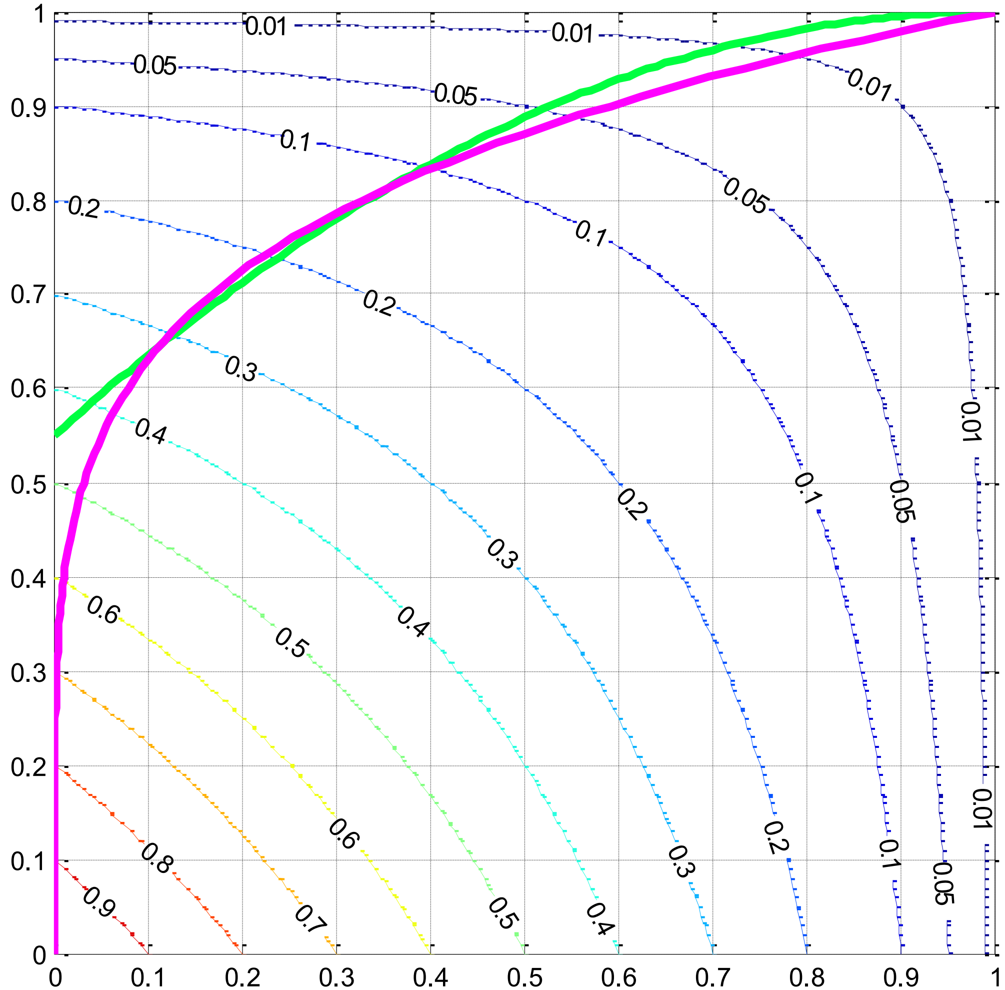
Intersecting Curves



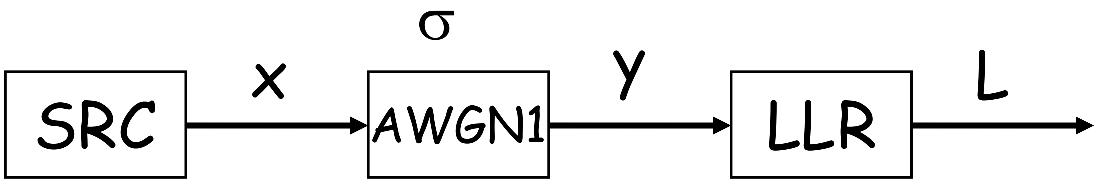
BER from EXIT Chart (BEC)

app = a + e

$$I(X;APP) = 1 - P_b = 1 - (1-I_A)(1-I_E)$$



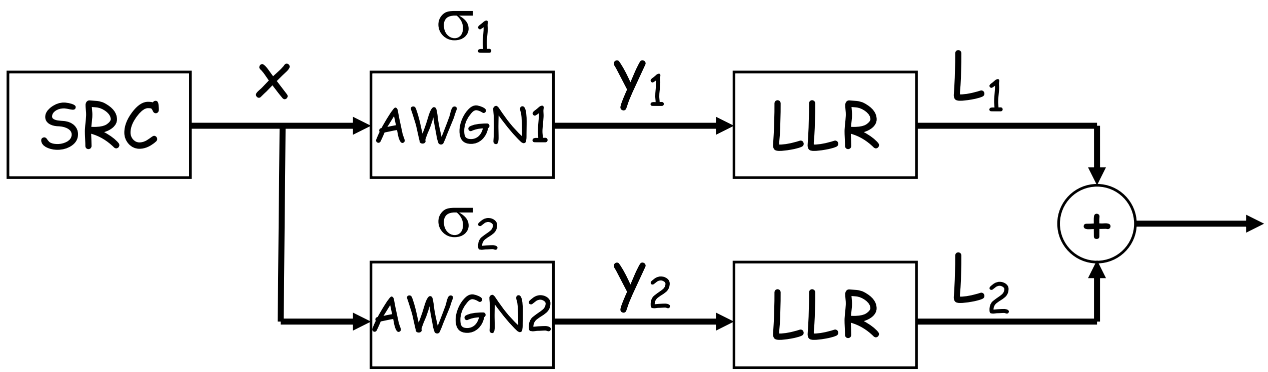
Information Combining AWGN



$$L = \frac{2y}{\sigma^2} \qquad \Rightarrow \qquad L \sim \mathcal{N}\left(\frac{2}{\sigma^2}, \frac{4}{\sigma^2}\right) = \mathcal{N}\left(\frac{\sigma_L^2}{2}, \sigma_L^2\right)$$

$$I(X;L) = 1 - \underbrace{\int_{-\infty}^{\infty} \frac{e^{-\frac{(\zeta - \sigma_L/2)^2}{2\sigma_L r}}}{\sqrt{2\pi}\sigma_L} \log_2\left(1 + e^{-\zeta}\right) d\zeta}_{J(\sigma_L)}$$

Information Combining AWGN



$$I_1 = I(X; L_1) = J(\sigma_{L1})$$

$$I_2 = I(X; L_2) = J(\sigma_{L2})$$

$$I(X; L_1 L_2) = J\left(\sqrt{\sigma_{L_1}^2 + \sigma_{L_2}^2}\right)$$

$$= J\left(\sqrt{J^{-1}(I_1)^2 + J^{-1}(I_2)^2}\right)$$

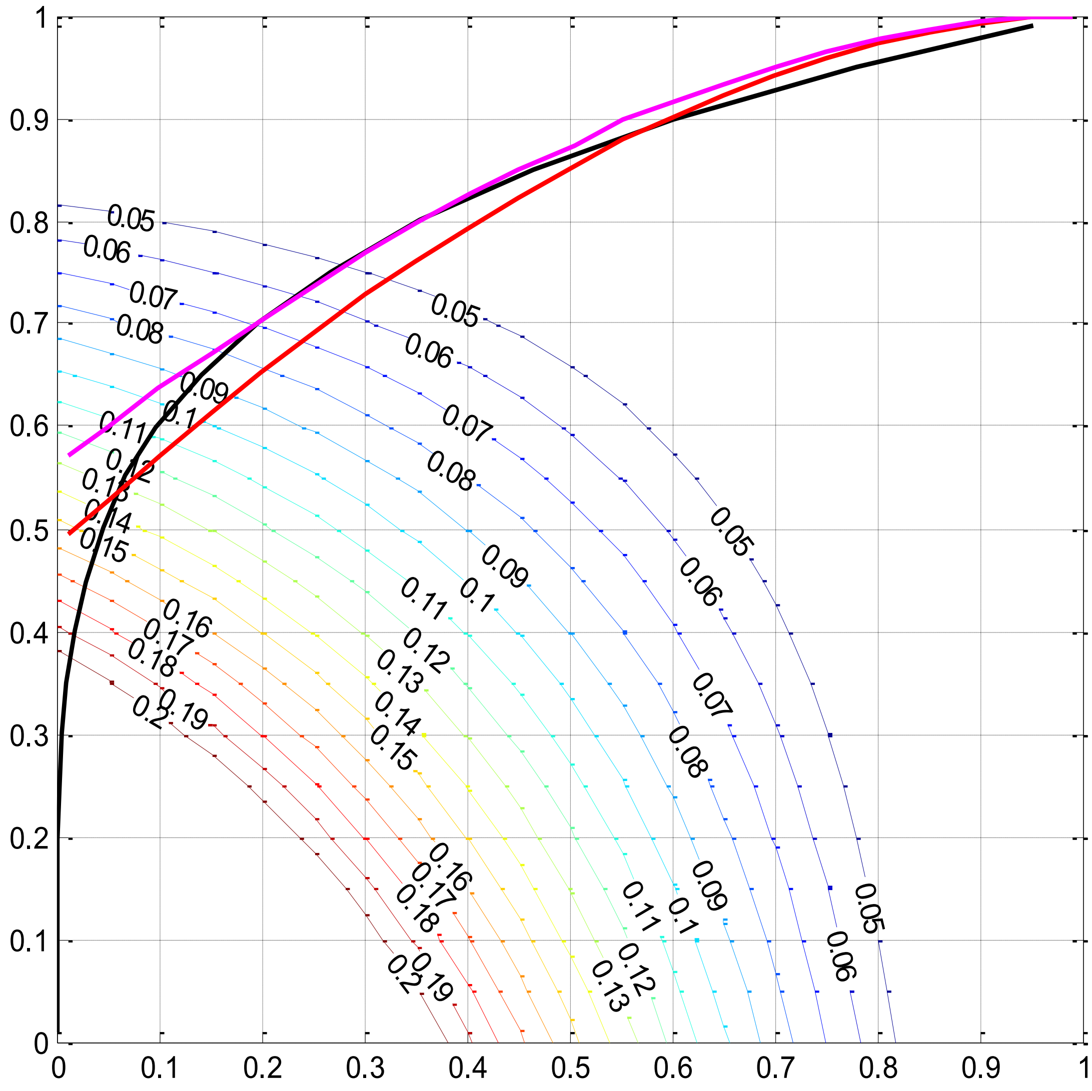
BER from EXIT Chart (AWGN)

E_b/N_0 = 0.0dB

P_b = 0.13

E_b/N_0 = 1.0dB

P_b = 0.07



Independent Observations

Messages received from the extrinsic channel are independent observations, which is only fulfilled if N $\rightarrow \infty$

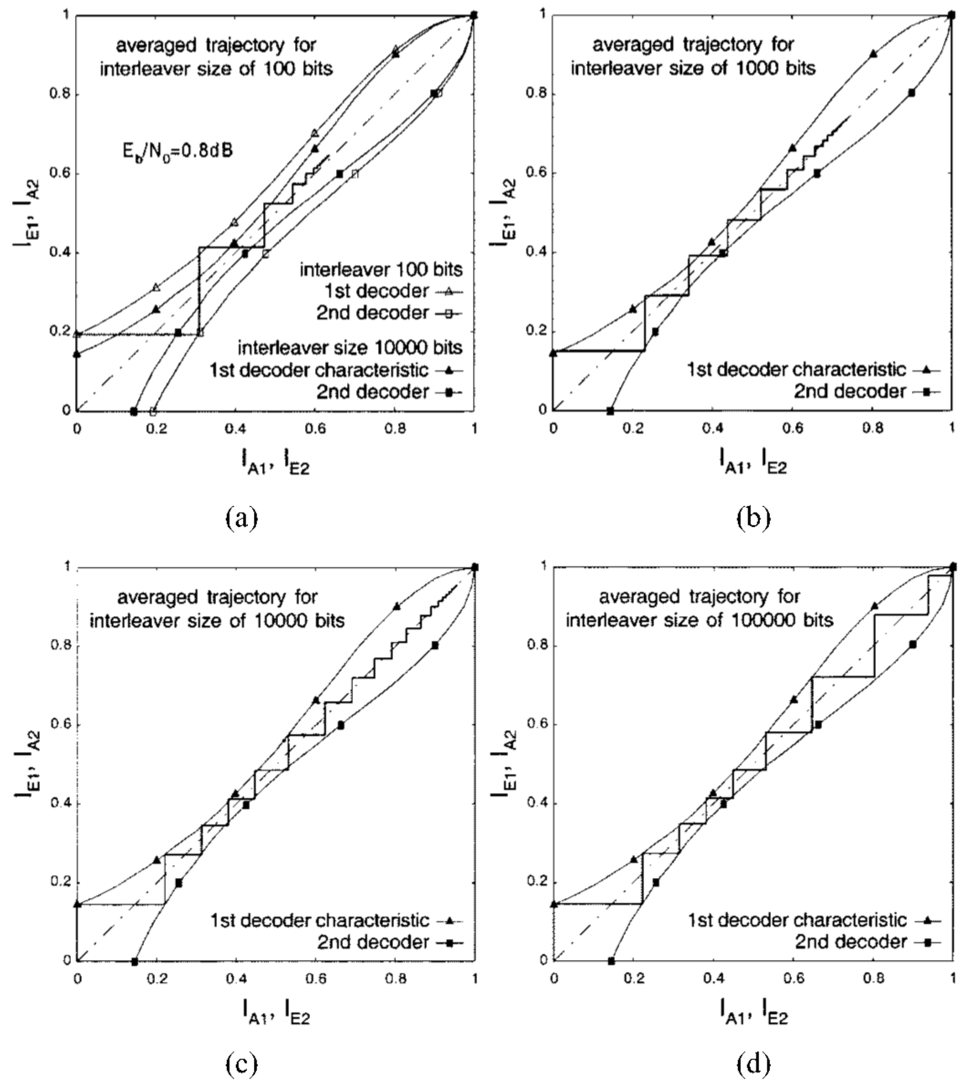
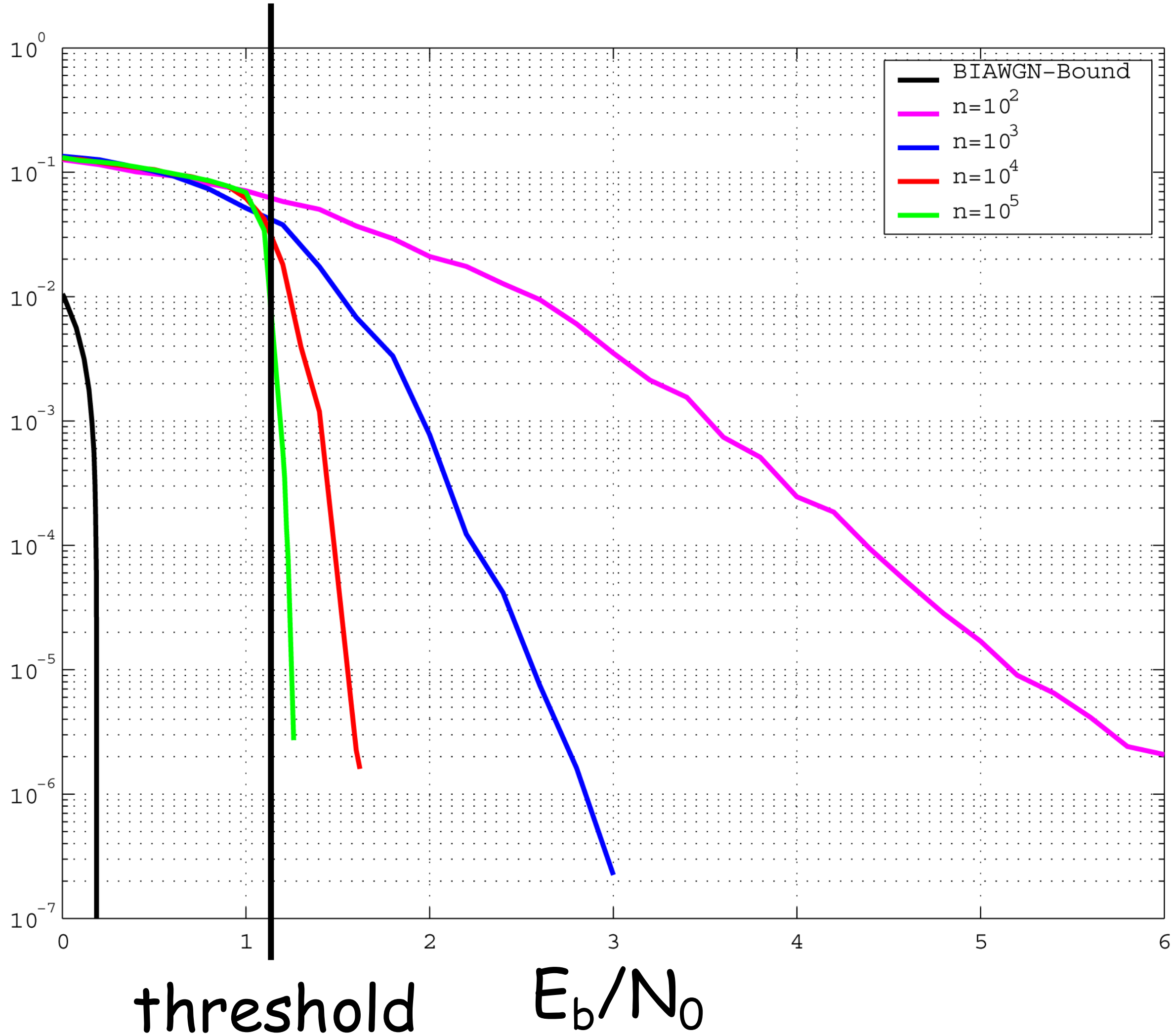


Fig. 8. Averaged decoding trajectories for different interleaver lengths; PCC rate 1/2 memory 4, $(G_r, G) = (023, 037)$; averaged over 10^8 information bits.

Statistics

We use **statistical quantities**, which P_b are only correct if $N \rightarrow \infty$



$$E_s = R \cdot E_b \qquad \sigma^2 = \frac{N_0}{2}$$

$$E_s := 1$$

$$\frac{E_b}{N_0} = \frac{E_s}{R 2\sigma^2} \Rightarrow \sigma^2 = \frac{1}{2R} \cdot \left(\frac{E_b}{N_0}\right)$$

Summary of Assumptions

- Messages received from the extrinsic channel are independent observations, which is only fulfilled if $N \rightarrow \infty$

- We use statistical quantities, which are only correct if $N \rightarrow \infty$

- We model extrinsic messages with an extrinsic channel. This can only be done exact for the BEC. The Gaussian assumption is an approximation.