3F1 Signals and Systems: Handout 11

The Discrete Fourier Transform (DFT)

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Outline

3 lectures on the Discrete Fourier Transform:

1. Theory of the Discrete Fourier Transform (DFT): revision of IB Paper 6 and a few more properties
2. Hardware implementations of the DFT: Fast Fourier Transform (FFT) algorithms
3. Applications: what you can do (or do better) with the FFT

The DTFT: theory vs. practice

The DTFT

$$X(\theta) = \sum_{k=-\infty}^{\infty} x_k e^{-jk\theta} \quad \text{where } \theta = \omega T = 2\pi f T$$

cannot be evaluated numerically

- it results in a continuous spectrum, i.e., a continuous function defined over the interval $[-f_S/2, f_s/2]$
- it requires an infinite sum

It could be computed numerically if we made the following simplifications:

- evaluate it only at N discrete frequencies on the unit circle (where N can be made as large as desired). This is a "sampled spectrum".
- compute only the first N terms of the sum, i.e., for the "truncated" sequence: $\{x_k\}_{0 \leq k \leq N-1}$ (this corresponds to a multiplication with a rectangular window)

The Discrete-Time Fourier Transform (DFT)

$$X_n = \sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}kn}$$

for $0 \leq n \leq N-1$.

- Can be computed numerically
- If we extended the range $0 \leq n \leq N-1$ to all integers n, the formula above still works and we'd obtain an N-period spectrum because

$$e^{-j\frac{2\pi}{N}k(n+aN)} = e^{-j\frac{2\pi}{N}kn}$$

  for any a.
- The transform now maps a finite-length sequence $\{x_k\}_{0 \leq k \leq N-1}$ to a finite-length sequence: $\{X_n\}_{0 \leq n \leq N-1}$
- $\longrightarrow$ **these are vectors!** (but note indexation 0 to $N-1$ rather than 1 to N)

The DFT as a matrix operation

We can re-write the DFT as a matrix-vector operation:

$$\begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ \vdots \\ X_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{-j\frac{2\pi}{N}} & \dots & e^{-j\frac{2\pi}{N}(N-1)} \\ 1 & e^{-j\frac{2\pi}{N}2} & \dots & e^{-j\frac{2\pi}{N}2(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\frac{2\pi}{N}(N-1)} & \dots & e^{-j\frac{2\pi}{N}(N-1)^2} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{N-1} \end{bmatrix}$$

This is tedious... It becomes much easier to visualise if we define the notation: $W_N = e^{-j\frac{2\pi}{N}}$.

The DFT matrix as a Vandermonde matrix

Using this notation, the DFT matrix becomes:

$$\mathbf{F} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & W_N & W_N^2 & \dots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \dots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \dots & W_N^{(N-1)^2} \end{bmatrix}$$

This is called a "Vandermonde" matrix with parameters $1, W_N, W_N^2, \dots, W_N^{N-1}$ and has interesting properties. For example, its determinant is easy to compute (but we won't use this.)

Properties of W_N

$$W_N = e^{-j\frac{2\pi}{N}}$$

- None of the powers $1, 2, \ldots, N-1$ of W_N are equal to one, i.e., $W_N, W_N^2, W_N^3, \ldots, W_N^{N-1} \neq 1$
- The $N-th$ power of W_N is equal to one, i.e., $W_N^N = W_N^0 = 1$
- W_N is called a primitive N-th root of unity
- For complex numbers, primitive roots of unity exist for every N, i.e., either $e^{j\frac{2\pi}{N}}$ of $e^{-j\frac{2\pi}{N}}$
- There is no particular reason why $W_N = e^{-j\frac{2\pi}{N}}$ was chosen. The minus is just a matter of preference and in line with the convention when going from time to frequency domain

Other DFTs

- The DFT we saw operates in the **complex** domain
- In the **real** domain, the only primitive N-th root of unity that exists is $W_2 = -1$ for $N = 2$. Hence, the only DFT that exists for real numbers has length $N = 2$:

$$\mathbf{F}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- Although this may seem trivial, it is in fact in widespread use and forms the basis of the Walsh-Hadamard transform, a multi-dimensional 2-point DFT that has many applications (but we won't study it)
- DFTs can be defined for other sets of numbers. In 4F5 when discussing error correction coding, we use a DFT defined for arithmetic modulo a prime number p ("Galois fields" of order p) and other finite sets of order $q = p^m$ with suitably defined arithmetic operations

Cyclic properties of the powers of W_N

Any integer k can be written as

$$k = qN + r$$

with $0 \leq r < N$, where q is called the **quotient** and r is called the **remainder** and denoted

$$r = R_N(k)$$

(this is known as "Euclid's division theorem")
We can write

$$W_N^k = W_N^{qN+r} = (W_N^N)^q \cdot W_N^r = 1^q \cdot W_N^r = W_N^r$$

Hence, we have established that

$$W_N^k = W_N^{R_N(k)}$$

$\rightarrow$ The DFT/Vandermonde matrix can be re-written so that powers of W_N higher than $N-1$ are expressed as powers between 0 and $N-1$.

Constructing the DFT matrix

The structure of the DFT matrix is as follows:

$$\mathbf{F} = \begin{bmatrix} 1 & 1 & 1 \dots & \text{(Row 0: } N \text{ ones)} \\ 1 & W_N & W_N^2 \dots & \text{(Row 1: powers 0 to } N-1 \text{ of } W_N\text{)} \\ 1 & W_N^2 & W_N^4 \dots & \text{(Row 2: every } 2^{\text{nd}} \text{ element of Row 1)} \\ 1 & W_N^3 & W_N^6 \dots & \text{(Row 3: every } 3^{\text{rd}} \text{ element of Row 1)} \\ \dots & \dots & \dots & \\ 1 & W_N^{N-1} & W_N^{N-2} \dots & \text{(Row } N-1\text{: Row 1 backward)} \end{bmatrix}$$

where "every k^{th} element" is understood cyclically, e.g., if you are in position $N-2$ and you are writing every second element, the next element is position $R_N(N-2+2) = 0$ which is $W_N^0 = 1$.
The last row is "every $(N-1)^{\text{th}}$ element" which is the same as going backward from position 0.

Examples of DFT matrices

- $\mathbf{F}_2 = \begin{bmatrix} 1 & 1 \\ 1 & W_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & e^{-j\pi} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (real!)
- $\mathbf{F}_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3 & W_3^2 \\ 1 & W_3^2 & W_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \\ 1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{2\pi}{3}} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{j\frac{2\pi}{3}} \\ 1 & e^{j\frac{2\pi}{3}} & e^{-j\frac{2\pi}{3}} \end{bmatrix}$
- $\mathbf{F}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4 & W_4^2 & W_4^3 \\ 1 & W_4^2 & 1 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-j\frac{\pi}{2}} & e^{-j\pi} & e^{-j\frac{3\pi}{2}} \\ 1 & e^{-j\pi} & 1 & e^{j\pi} \\ 1 & e^{-j\frac{3\pi}{2}} & e^{-j\pi} & e^{-j\frac{\pi}{2}} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$
- $\mathbf{F}_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & W_5 & W_5^2 & W_5^3 & W_5^4 \\ 1 & W_5^2 & W_5^4 & W_5 & W_5^3 \\ 1 & W_5^3 & W_5 & W_5^4 & W_5^2 \\ 1 & W_5^4 & W_5^3 & W_5^2 & W_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{5}} & e^{-j\frac{4\pi}{5}} & e^{-j\frac{6\pi}{5}} & e^{-j\frac{8\pi}{5}} \\ 1 & e^{-j\frac{4\pi}{5}} & e^{-j\frac{8\pi}{5}} & e^{-j\frac{2\pi}{5}} & e^{-j\frac{6\pi}{5}} \\ 1 & e^{-j\frac{6\pi}{5}} & e^{-j\frac{2\pi}{5}} & e^{-j\frac{8\pi}{5}} & e^{-j\frac{4\pi}{5}} \\ 1 & e^{-j\frac{8\pi}{5}} & e^{-j\frac{6\pi}{5}} & e^{-j\frac{4\pi}{5}} & e^{-j\frac{2\pi}{5}} \end{bmatrix}$
- Clearly there is some number-theoretic "magic" dictating whether there are repetitions in the row (e.g. $\mathbf{F}_4$) or not (e.g. $\mathbf{F}_3$ and $\mathbf{F}_5$) that depends on whether N is prime.
- Next handout, we will learn about algorithms that exploit the structure of $\mathbf{F}_N$ when N is not prime

Inner (dot) products for complex vectors

- The dot product for real vectors is $\vec{u}.\vec{v} = \sum_{k=0}^{N-1} u_k v_k$
- We can use the dot product to compute the magnitude squared: $\|\vec{u}\|^2 = \vec{u}.\vec{u}$
- For complex vectors, using the formula above would result in a complex magnitude. A vector's magnitude is a "norm" (the Euclidean, or L^2 norm in this case) and should always be a real non-negative number. Complex magnitudes are not allowed.
- The dot product for complex vectors is defined using complex conjugation as follows:

$$\vec{u}.\vec{v} = \sum_{k=0}^{N-1} u_k v_k^\star$$

  where $v_k^\star$ denotes the complex conjugate of v_k.
- We now obtain a real-valued non-negative squared magnitude: $\|\vec{u}\|^2 = \sum_{k=0}^{N-1} u_k u_k^\star = \sum_{k=0}^{N-1} |u_k|^2$

Sums of columns of $\mathbf{F}$

- Note first that

$$(W_N^k)^\star = \left(e^{-j\frac{2\pi}{N}k}\right)^\star = W_N^{-k} = W_N^{N-k}$$

- Furthermore, note that for $m \neq 0$,

$$\sum_{k=0}^{N-1} W_N^{km} = \frac{1-(W_N^m)^N}{1-W_N} = \frac{1-W_N^{R_N(mN)}}{1-W_N} = \frac{1-1}{1-W_N} = 0$$

- We used the sum from 0 to $N-1$ of a geometric sequence formula. Another way to see this is that the W_N^{km} for $k = 0, 1, \ldots, N-1$ are symmetrically placed complex numbers on the unit circle so their sum must be 0.
- In other words, the sum of any column of $\mathbf{F}$ except column 0 is zero. The sum of column 0 is

$$\sum_{k=0}^{N-1} 1 = N.$$

Orthogonality of $\mathbf{F}$

- Let $\vec{f}_\ell$ be the ℓ-th column of a DFT matrix $\mathbf{F}$. For $\ell > m$,

$$\vec{f}_\ell . \vec{f}_m = \sum_{k=0}^{N-1} W_N^{\ell k}(W_N^{mk})^\star = \sum_{k=0}^{N-1} W_N^{(\ell-m)k}$$

which is the sum of the $(\ell - m)$-th column, hence 0 if $\ell > m$.
- Setting $\ell = m$ in the above gives the sum of column zero, and hence $\vec{f}_\ell . \vec{f}_\ell = N$
- We have shown that the columns of $\mathbf{F}$ are orthogonal and that their magnitude is $\sqrt{N}$. The same applies to the rows because $\mathbf{F}$ is symmetric.
- The DFT is an orthogonal basis transform! (with a scaling factor of N)
- Some textbooks define the DFT with a factor of $1/\sqrt{N}$ to make $\mathbf{F}$ an orthonormal matrix (a rotation matrix!)

The inverse of F

- For a complex matrix $\mathbf{A}$ let $\mathbf{A}^\star$ denote its conjugate (Hermitian) transpose
- The orthogonality of $\mathbf{F}$ makes it easy to determine its inverse:

$$\mathbf{F}^{-1} = \frac{1}{N}F^\star \quad \text{because} \quad \frac{1}{N}\mathbf{F}\mathbf{F}^\star = \mathbf{I}_N$$

  where $\mathbf{I}_N$ denotes the $N \times N$ identity matrix.
- Note that transposition was not essential in determining the inverse, because $\mathbf{F}$ is symmetric. Hence all we need to do is conjugate the entries of $\mathbf{F}$ to get its inverse matrix, and hence the inverse DFT
- Since $(W_N^k)^\star = W_N^{N-k}$, the inverse matrix $\mathbf{F}^{-1} = \frac{1}{N}\mathbf{F}^\star$ has the same row 0 as $\mathbf{F}$ (because $1^\star = 1$) and its rows 1 to $N-1$ are the corresponding rows of $\mathbf{F}$ in reverse order, counting down from $N-1$ to 1.

The inverse DFT and examples

- We have re-derived the inverse DFT formula

$$x_k = \frac{1}{N}\sum_{n=0}^{N-1} X_n W_N^{-kn} = \frac{1}{N}\sum_{n=0}^{N-1} X_n e^{j\frac{2\pi}{N}kn}$$

- $\mathbf{F}_2^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
- $\mathbf{F}_3^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} \\ 1 & e^{j\frac{4\pi}{3}} & e^{j\frac{2\pi}{3}} \end{bmatrix}$
- $\mathbf{F}_4^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$
- $\mathbf{F}_5^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & e^{j\frac{2\pi}{5}} & e^{j\frac{4\pi}{5}} & e^{j\frac{6\pi}{5}} & e^{j\frac{8\pi}{5}} \\ 1 & e^{j\frac{4\pi}{5}} & e^{j\frac{8\pi}{5}} & e^{j\frac{2\pi}{5}} & e^{j\frac{6\pi}{5}} \\ 1 & e^{j\frac{6\pi}{5}} & e^{j\frac{2\pi}{5}} & e^{j\frac{8\pi}{5}} & e^{j\frac{4\pi}{5}} \\ 1 & e^{j\frac{8\pi}{5}} & e^{j\frac{6\pi}{5}} & e^{j\frac{4\pi}{5}} & e^{j\frac{2\pi}{5}} \end{bmatrix}$

Conjugate symmetry

- In a DFT matrix $\mathbf{F}$,
  $$\vec{f}_n = \vec{f}^{\star}_{N-n},$$
  i.e., columns (and rows) 1 to $\lceil N/2 - 1 \rceil$ come in complex conjugate pairs, while column 0 is real (all ones) and column $N/2$ (for even N) is also real (alternating 1s and -1s).
- Hence, **if the input vector $\vec{x}$ is real**, the frequency domain output vector $\vec{X}$ exhibits conjugate symmetry
  $$X_n = X^{\star}_{N-n}.$$
  In other words, $\begin{cases} |X_n| = |X_{N-n}| \\ \angle X_n = -\angle X_{N-n} \\ X_0 \text{ is real} \\ X_{N/2} \text{ is real (for even } N) \end{cases}$
- Conversely, if an input time-domain vector obeys the conjugate symmetry property, its DFT is real.

Conjugate symmetry example

Exercise: connect the time domain / frequency domain DFT pairs

Time domain	**Frequency domain**
[1, 2, 3, 4, 4, 3, 2, 1]	[10, 2, -6, 3.66, -6, 2, -6, -7.66]
[1+j, 2+j, 3+j, 4+j, 0, 0, 0, 0]	[20, -5.83-2.41j, 0, -0.17-0.41j, 0, -0.17+0.41j, 0, -5.83+2.41j]
[-1, 1, 2, 3, -1, 3, 2, 1]	[10+4j, 2-6.24j,-2+2j, 2.83-0.24j,-2, 2+2.24j, -2-2j,-2.83+8.24j]
[-1, 1+j, 2+j, 3+j, -1, 3-j, 2-j, 1-j]	[10, -2.83, -6, 2.83, -6, 2.83, -6, -2.83]

Circular (cyclic) convolution

- The circular convolution of vectors $\vec{x}$ and $\vec{y}$

$$\vec{z} = \vec{x} \circledast \vec{y}$$

is defined as

$$z_k = \sum_{m=0}^{N-1} x_m y_{R_N(k-m)},$$

i.e., like a regular discrete convolution but the index of the second vector is taken "circularly" between 0 and $N-1$ by taking $k-m$ "modulo N"

- Like convolution, the circular convolution is commutative

$$\vec{x} \circledast \vec{y} = \vec{y} \circledast \vec{x}$$

as can easily be shown by substituting the index in the summation.

Circular convolution property of the DFT

- Like all the transforms we studied, the DFT has a convolution property
- Its convolution property applies to the circular convolution
- This can be a benefit (when it's the circular convolution you want) or a drawback (when you want a linear convolution). We will study examples of both.
- For the DFT, this is an essential property. We will soon study algorithms for efficient computation of the DFT. Convolutions are universally computed via the DFT in countless applications from signal processing to numerical calculus

Circular convolution property

Circular convolution in the time domain is equivalent to point-wise multiplication in the frequency domain, and vice-versa.

Proof the circular convolution property

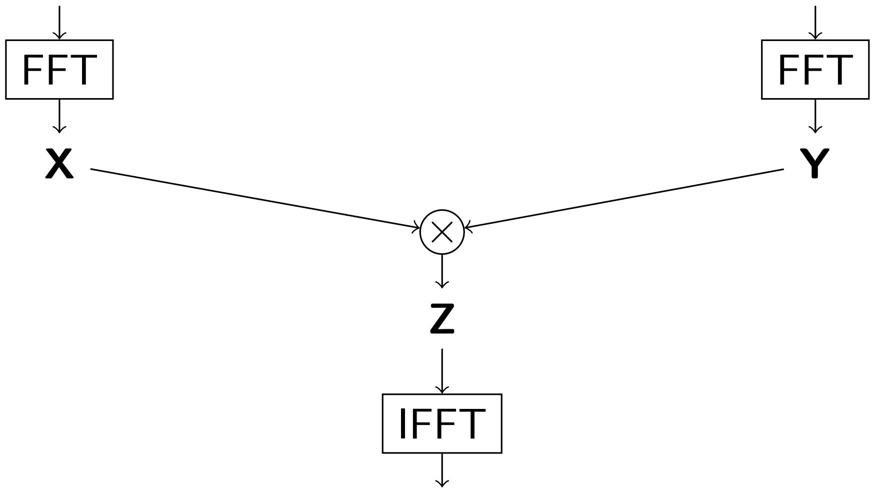
Let $\vec{z} = \vec{x} \circledast \vec{y}$ and let $\vec{Z}$ be the DFT of $\vec{z}$. We have

$$
\begin{aligned}
Z_n &= \sum_{k=0}^{N-1} z_k W_N^{kn} = \sum_{k=0}^{N-1} \left(\sum_{m=0}^{N-1} x_m y_{R_N(k-m)} \right) W_N^{kn} \\
&= \sum_{m=0}^{N-1} x_m \sum_{k=0}^{N-1} y_{R_N(k-m)} W_N^{kn} \quad \longrightarrow k' = R_N(k-m) \\
&= \sum_{m=0}^{N-1} x_m \sum_{k'=0}^{N-1} y_{k'} W_N^{nR_N(k'+m)} = \sum_{m=0}^{N-1} x_m \sum_{k'=0}^{N-1} y_{k'} W_N^{n(k'+m)} \\
&= \sum_{m=0}^{N-1} x_m W_N^{nm} \sum_{k'=0}^{N-1} y_{k'} W_N^{nk'} = X_n Y_n
\end{aligned}
$$

The proof of the reverse property follows the same lines.

Example of convolution

$\mathbf{x} = [0, 1, 1, 0, 0, 0, 0]$ $\qquad$ $\mathbf{y} = [0, 1, 1, 0, 0, 0, 0, 0]$



$\mathbf{z} = [0, 0, 1, 2, 1, 0, 0, 0]$

- in the example, $\mathbf{x} = \mathbf{y}$
- $\mathbf{Y} = \mathbf{X} =$ [2, 0.71-1.71j, -1-1j, -0.71+0.29j, 0,-0.71-0.29j, -1+1j, 0.71+1.71j]

$$
\begin{aligned}
\mathbf{Z} &= [X_0 Y_0, X_1 Y_1, X_2 Y_2, X_3 Y_3, X_4 Y_4, X_5 Y_6, X_7 Y_7] \\
&= [X_0^2, X_1^2, X_2^2, X_3^2, X_4^2, X_5^2, X_6^2, X_7^2]
\end{aligned}
$$

Time and frequency shift properties

- Like other transforms, the DFT has time and frequency shift properties. However, for the DFT these are **cyclic** time and frequency shifts
- Consider the input shifted by d cyclically, i.e., $x'_k = x_{R_N(k+d)}$
- The DFT is

$$
\begin{aligned}
X'_n &= \sum_{k=0}^{N-1} x'_k W_N^{kn} = \sum_{k=0}^{N-1} x_{R_N(k+d)} W_N^{kn} \quad \longrightarrow k' = R_N(k-m) \\
&= \sum_{k'=0}^{N-1} x_{k'} W_N^{nR_N(k'-d)} = \sum_{k'=0}^{N-1} x_{k'} W_N^{k'n-dn} \\
&= W_N^{-dn} \sum_{k'=0}^{N-1} x_{k'} W_N^{k'} = W_N^{-dn} X_n
\end{aligned}
$$

- Similarly, if $X'_n = X_{R_N(n+d)}$, then $x'_k = W_N^{dk} x_k$

DFT properties: summary

- the DFT has similar properties to the DTFT, the z transform, the Fourier transform, etc.
  - convolution property and reverse convolution property
  - time and frequency shift properties
  - conjugate symmetry for real time sequence / real spectrum
- all properties are cyclic (circular)
- as the DFT is the only transform you're likely to use in practice (Python, MATLAB, etc.) it's important that you familiarise yourself with the details of these cyclic properties

Parseval

- For an orthonormal basis transform, the dot product expressed in the new set of coordinates remains unchanged
- The DFT as we defined it is orthogonal but the new basis vectors have a magnitude of $\sqrt{N}$
- Hence, if $\vec{X} = \mathbf{F}\vec{x}$ and $\vec{Y} = \mathbf{F}\vec{y}$,

$$\vec{X}.\vec{Y} = \sum_{n=0}^{N-1} X_n Y_n^{\star} = N\vec{x}.\vec{y} = N\sum_{k=0}^{N-1} x_k y_k^{\star}$$

- In particular,

$$\|\vec{x}\|^2 = \sum_{k=0}^{N-1} |x_k|^2 = \frac{\|\vec{X}\|^2}{N} = \frac{1}{N}\sum_{n=0}^{N-1} |X_n|^2$$

Relating the DFT to the frequencies of the sampled signal

- We began by presenting the DFT as an approximation by truncation and sampling of the continuous DTFT spectrum
- The N frequency readings obtained from the DFT are "sampled" frequencies between 0 and 2π in normalised frequency (i.e., 2π corresponds to the sampling frequency f_s.)
- X_0 corresponds to frequency 0 ("D.C." or bias): it is zero if the sum (or average) of $\vec{x}$ is zero
- For even N, $X_{N/2}$ corresponds to π, i.e., $f_2/2$
- Position 1 to $\lceil N/2 - 1 \rceil$ correspond to equally spaced positive frequency points between 0 and π, i.e., $f_s/2$
- Frequencies $N-1$ down to $\lfloor N/2 + 1 \rfloor$ correspond to "negative" frequencies between 0 and $-\pi$, i.e., $-f_s/2$
- Python and Matlab have a command "fftshift" that rotate the vector $\vec{X}$ around its middle so that the "negative" frequencies are displayed to the left of the zero frequency

DFT vs. DTFT

- Ideally, one would like the value X_n to be the integral of the DTFT spectrum over the interval $[\frac{2\pi}{N}(n-1/2), \frac{2\pi}{N}(n+1/2)]$. Unfortunately, it isn't!
- Truncation of the sequence $\{x_k\}_{k\geq 0}$ to go from DTFT to DFT is equivalent to a multiplication of the sequence by a rectangular window of length N.
- As we observed when studying the truncation of the impulse response of an ideal filter, this is equivalent to a convolution of the spectrum with a function $W(e^{j\theta}) = \sin\left(\frac{\theta}{2}(N+1)\right)/\sin\left(\frac{\theta}{2}\right)$
- Hence, the value of X_n depends on the whole DTFT spectrum by convolution, with a "main lobe" around $\theta = \frac{2\pi}{N}n$
- There is "leakage" into neighbouring frequency bins
- The matching of DFT to DTFT spectrum can be improved by using window methods just like for filtering.

Outline

3 lectures on the Discrete Fourier Transform:

1. ~~Theory of the Discrete Fourier Transform (DFT): revision of IB Paper 6 and a few more properties~~
2. Hardware implementations of the DFT: Fast Fourier Transform (FFT) algorithms
3. Applications: what you can do (or do better) with the FFT

You can now do questions 1 to 11 in EP2. For Question 12, you'll need to wait 2 more lectures.